CivilEng

Article

Experimental Study on the Minimum Required Specimen Width to Maximum Particle Size Ratio in Direct Shear Tests

Akram Deiminiat , Li Li * and Feitao Zeng

Research Institute on Mining and Environment, Department of Civil, Geological and Mining Engineering, Polytechnique Montreal, Montreal, QC H3T 1J4, Canada; akram.deiminiat@polymtl.ca (A.D.); feitao.zeng@polymtl.ca (F.Z.)
* Correspondence: li.li@polymtl.ca

Abstract: Conducting laboratory direct shear tests on granular materials is a common practice in geotechnical engineering. This is usually done by following the ASTM D3080/D3080M-11 (hereafter named ASTM), which stipulates a minimum required value of 10 for specimen width (W) to the maximum particle size ($d_{\max}$) ratio. Recently, a literature review performed by the authors showed that the minimum required $W/d_{\max}$ ratio given in the ASTM is not large enough to eliminate the specimen size effect (SSE). The minimum required $W/d_{\max}$ ratio of ASTM needs to be revised. In this study, a critical analysis is first made on existing data in order to identify the minimum required $W/d_{\max}$ ratio. The analysis shows that more experimental data obtained on specimens having $W/d_{\max}$ ratios between 10 and 50 are particularly necessary. To complete this need, a series of direct shear tests were performed on specimens having different $d_{\max}$ by using three shear boxes of different dimensions. The results show once again that the minimum required $W/d_{\max}$ ratio of 10, defined in the ASTM, is not large enough to eliminate the SSE. Further analysis on these and existing experimental results indicates that the minimum required $W/d_{\max}$ ratio to remove the SSE of friction angles is about 60. These results along with the limitations of this study are discussed.

Keywords: shear strength; direct shear tests; friction angle; specimen size effect; minimum required specimen sizes; ASTM D3080/D3080M-11

Citation: Deiminiat, A.; Li, L.; Zeng, F. Experimental Study on the Minimum Required Specimen Width to Maximum Particle Size Ratio in Direct Shear Tests. *CivilEng* **2022**, *3*, 66–84. https://doi.org/10.3390/civileng3010005

Academic Editors: João Castro-Gomes, Cristina Fael and Miguel Nepomuceno

Received: 15 November 2021
Accepted: 19 January 2022
Published: 21 January 2022

1. Introduction

The direct shear test is a very old but still regularly used method to determine the shear strength of geomaterials [1–16]. For a given project, one usually needs to take samples from the project site. Specimens can then be prepared in a laboratory with a small portion of the samples. For the convenience of laboratory tests, tested specimens are preferred to be as small as possible. However, the method imposes a shear plane through the tested material between the upper and lower half parts of the shear box. When the specimen size is too small, the effect of individual particles along and near the imposed shearing plane can be amplified in terms of rotation, crushing, shearing, and dilation during direct shear tests. The distributions of stresses and strains can be non-uniform along the shearing plane [17–20]. The measured shear strength may be overestimated and not representative of that of the tested material in field conditions where the volume of the tested material can be very large [21]. The specimen size of tested material should not be too small.

Increasing the specimen size of the tested material reduces the effect of individual large particles and boundary effects associated with the stiff shear box walls. The measured shear strength can be closer to that of the tested material in field conditions. When the specimen size is large enough, the measured shear strength can then become constant with any further increase in the specimen size. The variation of shear strength as function of specimen size is known as specimen size effect or specimen scale effect (SSE). Finding the large enough specimen size to avoid any SSE is also known as a problem of representative volume element size [22–24].

Over the years, a number of studies have been published on the SSE of direct shear tests. Most of them were realized by performing laboratory tests [8,13,14,25–34]. A few studies have been realized through numerical modeling [35–39]. Each method has advantages and limitations. With experimental investigation, the test results are direct, tangible, and more convincing as long as the tests are properly realized. Its limitations are also obvious: shear box and specimen sizes are limited. The reliability of test results depends on the representativeness of assumptions, implicitly or explicitly used in the result interpretation. The accessibility to observe the movements of particles or other mechanical responses of the tested specimen during shear tests is difficult, if not impossible [40,41]. With numerical modeling, the stress and deformation anywhere through the modeled specimen can be visualized. This allows an insight to the micromechanical behavior, which can, in turn, lead to a better understanding of the macro mechanical behavior of material under direct shear test condition. However, all the numerical models, whatever continuum or discrete with or without meshes, are established with some discussable assumptions. Model calibrations are always necessary against experimental results to obtain model parameters. The representativeness of constitutive model or particle size distribution of granular material used in the numerical models depends on the reliability of the experimental results used for parameter calibration. In addition, ensuring the stability and reliability of numerical results is another challenging issue, especially at the zones of soil-wall interfaces and high gradient stress or deformation.

In this study, focus is given on the experimental study of SSE in direct shear tests.

2. Previous Laboratory Tests on SSE of Direct Shear Tests

In the past, a large number of experimental studies have been published on the SSE of direct shear tests. As the objective of the present study is to identify the minimum required specimen sizes to avoid SSE in direct tests, instead of an exhaustive literature review, analyses are only made on a few published works.

Over the years, several standards have been proposed to specify the minimum required ratios between the specimen sizes and maximum particle size ($d_{\max}$) of tested material. Table 1 shows a few standards commonly used in geotechnical engineering with the specification of specimen sizes. For example, ASTM D3080/D3080M-11 [42] (hereafter named ASTM), the most used standard for direct shear tests all over the world, requires specimen width (W) to be at least 10 times $d_{\max}$. In addition, specimen width W should not be smaller than 50 mm. For fine particle materials such as clay, silt, and sand with $d_{\max}$ not larger than 1 mm, the requirement of $W \geq 50$ mm automatically results in $W/d_{\max} \geq 50$, a value largely exceeding the minimum required ratio. Respecting the standard of ASTM does not raise any problem of SSE. Problems appear when direct shear tests are needed with coarse particle materials such as rockfill and waste rocks.

Table 1. Standards of direct shear tests regarding maximum particle size ($d_{\max}$), specimen width (W) and thickness (T).

Standard	Required Shear Box Size			Maximum Allowed $d_{\max}$
	W (mm)	T (mm)	W/T	
ASTM D3080/D3080M-11 [42]	≥50	≥13	≥2	Min{T/6, W/10}
Eurocode 7 [43]	Not specified	Not specified	Not specified	T/10
AS 1289.6.2.2 [44]	Not specified	≥12.5	Not specified	T/6
BS 1377–7 [45]	60 100 305	20 25 150	3 4 ≈2	2 mm 2.5 mm 15 mm–20 mm

Rockfill and chemically inert waste rocks are widely used to construct geotechnical infrastructures such as rockfill dikes of hydraulic reservoirs, waste rock piles [46,47], tailings dams [48,49], waste rock inclusions in tailings storage facilities [50–53], and waste rock barricades in underground mines to maintain backfill slurry in mine stopes [54–57]. The design of these infrastructures requires the knowledge of the shear strength of rockfill or waste works. Direct shear tests are then needed on these materials. However, rockfill and waste rocks usually contain fine particles as small as silts and coarse particles as large as boulders. Using a standard direct shear test box (6 cm for most cases) with field materials and respecting the minimum required specimen sizes to $d_{\max}$ ratio are impossible. Scaling down techniques are thus employed by removing oversized particles during sample preparation to make laboratory tests possible [16,58–67]. For most cases, the $d_{\max}$ of the resulting samples can still be too large compared to any available standard or nonstandard direct shear test apparatus. The minimum required specimen size to $d_{\max}$ ratio of 10 [42,68] has thus been universally used in direct shear tests [11,16,62,69–74] even though the validity of this minimum required value of 10 has never been verified. In other words, it remains unclear if the SSE of direct shear tests is eliminated with specimens having a width to $d_{\max}$ ratio of 10.

Parsons [25] studied the SSE of a crushed quartz and Ottawa sand with a $d_{\max}$ value of approximately 0.841 mm. Direct shear tests were conducted with small square (60 mm × 60 mm), medium rectangular (120 mm × 100 mm) and large rectangular (120 mm × 200 mm) shear boxes. Loose samples were poured in the shear boxes with a spoon at a constant falling height. The surface of the samples was smoothed to a standard thickness. The measured friction angles are presented in Table 2 along with the corresponding values of $W/d_{\max}$. For the test results with the crushed quartz, it is difficult to evaluate if the minor diminution of friction angle is an exhibition of SSE or simply due to the precision of measurement, sample variation or shape effect. As the smallest $W/d_{\max}$ ratio of 71 is already much higher than 10, these results do not provide valuable information on the validity of the minimum required specimen size over d_{max} ratio of ASTM. For the test results with Ottawa sand, Table 2 indicates that the friction angle continues to change significantly as the $W/d_{\max}$ ratio increases from 71 to 143. These results tend to indicate that the minimum required ratios of ASTM are not large enough to eliminate the SSE even at $W/d_{\max}$ ratio of 71. However, this conclusion may not be strong enough because the test results may contain several uncertainties associated with the precision of measurement, sample variation from small to large box, and shape effect.

Table 2. Variation of friction angle (ϕ) obtained by Parson (1936) with different specimen sizes through direct shear tests; L is the length of shear box.

L (mm) × W (mm)	$W/d_{\max}$	ϕ (°)	
		Crushed Quartz	**Ottawa Sand**
60 × 60	71	31.5	31.0
120 × 100	119	31.1	29.6
200 × 120	143	30.7	28.5

Similar tests to those of Parsons [25] have been conducted by Palmeira and Milligan [28] with one single normal stress of 30 kPa on a sand with $d_{\max}$ = 1.2 mm by using small square (60 mm × 60 mm × 32 mm), medium rectangular (252 mm × 152 mm × 152 mm), and large square (1000 mm × 1000 mm × 1000 mm) shear boxes. All the specimens were prepared by applying a pluviation technique with a constant falling height. The test results are shown in Table 3. One sees that the friction angles of the specimens remain almost constant when $W/d_{\max}$ increases from 50 to 833, values much larger than the minimum required $W/d_{\max}$ ratio of ASTM. These results tend to indicate that a $W/d_{\max}$ ratio of 50 is large enough to remove the SSE, but cannot help to evaluate if the minimum required ratios of ASTM are large enough to eliminate the SSE. In addition, it is unclear if $W/d_{\max}$ = 50 is the minimum required ratio or can be smaller to remove the SSE.

Table 3. Friction angles of a sand with d_{max} = 1.2 mm, obtained by direct shear tests with three shear box sizes [28].

L (mm) × W (mm) × T (mm)	W/d_{max}	T/d_{max}	ϕ (°)
60 × 60 × 32	50	27	50.1
252 × 152 × 152	127	127	50.2
1000 × 1000 × 1000	833	833	49.4

Rathee [26] investigated the SSE on the measurement of friction angle by direct shear tests. One tested material was made of pure gravel and another was a mixture of sand and gravel with the same portion. A small (60 mm × 60 mm) and a large (300 mm × 300 mm) shear box were used (the heights of the shear boxes were not provided in Rathee [26]). For the pure gravel material, only the large shear box was used to measure the friction angles of different samples having different d_{max} prepared by applying a scaling down technique. These results cannot be used to evaluate the SSE because each sample having a distinct value of d_{max} represents a distinct material. The variation of friction angle results from combined effects associated with the variation of d_{max}, particle size distribution, and specimen size to d_{max} ratio. The employed methodology is inappropriate to investigate the SSE. For the sand-gravel mixture material with d_{max} = 6.3 mm, both the small and large shear boxes were used by Rathee (1981) to measure the friction angle by direct shear tests. The experimental results showed that the friction angle decreases by about 1.7° when the W/d_{max} ratio increases from 10 to 48. These results clearly indicate that the SSE is not eliminated with a W/d_{max} ratio of 10. The minimum required specimen size to d_{max} ratio defined by ASTM is invalidated. However, as there are no results with W/d_{max} smaller or larger than 48, it is unclear if the W/d_{max} ratio of 48 is large enough to eliminate SSE.

The influence of specimen size on friction angle was further studied by Cerato and Lutenegger [8], who conducted direct shear tests with a small (59.9 mm × 59.9 mm × 26.4 mm), a medium (101.6 mm × 101.6 mm × 40.6 mm) and a large (304.8 mm × 304.8 mm × 177.8 mm) shear box on a sand and a gravel with different d_{max} and relative densities. For the specimen of gravel with d_{max} value of 5 mm, a known quantity of sample was placed in each box and compacted to achieve the desired relative density. Table 4 shows the measured friction angles of the gravel compacted to different relative densities. The experimental results clearly indicate that the W/d_{max} ratios of 12 and 20 are not large enough to remove the SSE, invalidating once again the minimum required W/d_{max} ratio of 10 given in ASTM. It is, however, unclear if the W/d_{max} ratio of 61 is large enough to remove the SSE.

Table 4. Friction angles of gravel having a d_{max} value of 5 mm with different relative densities (D_r), obtained by direct shear tests with three shear box sizes [8].

W (mm) × W (mm) × T (mm)	W/d_{max}	T/d_{max}	ϕ (°)		
			Dr = 25%	Dr = 55%	Dr = 85%
59.9 × 59.9 × 26.4	12	5	42.0	44.5	45.5
101.6 × 101.6 × 40.64	20	8	36.5	41.0	43.0
304.8 × 304.8 × 177.8	61	36	34.0	40.2	42.0

Wu et al. [30] conducted direct shear tests on a sand with a d_{max} value of 0.42 mm with a small square (40 × 40 × 20 mm), a medium square (120 × 120 × 120 mm), a large square (300 × 300 × 300 mm) and a very large rectangular (800 × 500 × 600 mm) shear boxes. The minimum W/d_{max} ratio is 95. Among the numerous specimens, only two have close enough compactness and void ratio. The test results presented in Table 5 tend to indicate that the SSE is not eliminated even at W/d_{max} = 95. The minimum required W/d_{max} = 10, given in ASTM, is invalidated once again.

Table 5. Friction angles of the sand samples obtained by Wu et al. [30].

Void Ratio	W (mm) × W (mm) × T (mm)	W/d_{max}	T/d_{max}	ϕ (°)
0.654	40 × 40 × 20	95	48	45.8
0.659	120 × 120 × 120	285	285	41.8

Mirzaeifar et al. [32] performed direct shear tests with a small (60 mm × 60 mm × 16 mm), a medium (100 mm × 100 mm × 30 mm), and a large (300 mm × 300 mm × 180 mm) shear box. The tested material is sand with a d_{max} value of 1.3 mm. Three samples were prepared, the first one at a density of 1.5 g/cm^3, the second at 1.58 g/cm^3, and the third at 1.67 g/cm^3, respectively. Table 6 shows the friction angles obtained by the direct shear tests. The results tend to indicate that the W/d_{max} ratio of 46 is not large enough while a W/d_{max} ratio up to 77 may be necessary to remove the SSE. The minimum required W/d_{max} ratio of 10, stipulated by the standard of ASTM, is invalidated once again.

Table 6. Friction angles of sand at three different densities, obtained by direct shear tests with three different size shear boxes [32].

W (mm) × W (mm) × T (mm)	W/d_{max}	T/d_{max}	ϕ (°)		
			at 1.5 (g/cm^3)	at 1.58 (g/cm^3)	at 1.67 (g/cm^3)
60 × 60 × 16	46	12	35.4	37.9	39.7
100 × 100 × 30	77	23	33.3	34.5	35.2
300 × 300 × 180	231	138	32.6	34.0	34.5

Ziaie Moayed et al. [34] studied the SSE on the friction angles of silty sand with a d_{max} value of 0.8 mm. The samples were prepared by mixing sand with different silt contents (Sand, 0%; Silty sand I, 10%; Silty sand II, 20%; Silty sand III, 30%). Small (60 mm × 60 mm × 24.5 mm), medium (100 mm × 100 mm × 35 mm), and large (300 mm × 300 mm × 154 mm) shear boxes were used to perform direct shear tests. Table 7 shows the friction angles obtained by only considering the direct shear test results of Ziaie Moayed et al. [34] with the normal stresses of 109, 163, and 218 kPa; the experimental results obtained with the normal stresses of 327 and 436 kPa were ignored due to the lack of tests using the large shear box with these two normal stresses. The friction angles were obtained by applying the linear fitting technique without imposing the straight lines passing through the origin in the shear stress–normal stress plane, resulting in apparent cohesions varying from 11 to 27 kPa with the small shear box tests and apparent cohesion ranging from −0.4 to 5 kPa with the large shear box tests. The results of the sand and silty sand I tend to show that the W/d_{max} ratio of 75 is not large enough to eliminate SSE, invalidating once again the minimum required W/d_{max} ratio of 10, stipulated by the standard of ASTM. With the test results of silty sand III, the W/d_{max} ratio of 75 seems to be large enough to avoid SSE. These results do not help to validate or invalidate the minimum required W/d_{max} ratio of 10, stipulated by the standard of ASTM.

More studies on the SSE of geomaterials can be found in the literature. For instance, Dadkhah et al. [75] studied the SSE of clayey sand. The SSE was clearly showed and the minimum required W/d_{max} ratio of 10 is invalidated again. However, the experimental results are difficult to be exploited because the values of d_{max} were not explicitly given (4.75 mm in a table and 10 mm in a figure of particle size distribution curves). In addition, it is unclear if the tested specimens were made of remoulded or unremoulded samples and how the designated densities of specimens were achieved. These results are thus not addressed further.

Table 7. Friction angles of sand and silty sands, obtained by only considering the direct shear test results of Ziaie Moayed et al. [34] with the normal stresses of 109, 163, and 218 kPa.

W (mm) × W (mm) × T (mm)	$W/d_{\max}$	$T/d_{\max}$	ϕ (°)			
			Sand [1]	Silty Sand I [2]	Silty Sand II [3]	Silty Sand III [4]
60 × 60 × 24.5	75	30.6	43.9	39.7	34.4	30.4
100 × 100 × 35	125	43.8	39.0	31.4	31.2	31.0
300 × 300 × 154	375	192.5	34.9	33.7	30.8	31.3

Note: For sand and silty sand II, the peak shear stresses at the three normal stresses are taken from the stress-displacement curves to obtain the friction angles because the use of the peak shear stresses from Figure 10 of Ziaie Moayed et al. [34] may result in negative cohesion. For silty sand I and III, no stress-displacement curves were given in [34]. [1] Results based on Figure 5 of Ziaie Moayed et al. [34]; [2] Results based on Figure 10b of Ziaie Moayed et al. [34]; [3] Results based on Figure 6 of Ziaie Moayed et al. [34]; [4] Results based on Figure 10d of Ziaie Moayed et al. [34].

More recently, MotahariTabari and Shooshpasha [14] studied the SSE of a coarse grain material in direct shear tests. Similarly to Rathee [26], the test results involve the effects of several influencing factors such as $d_{\max}$, fine and gravel contents and density. These results can neither be included in this study.

Table 8 shows a summarization of the previous analyses on the existing experimental results obtained to investigate the SSE. None of the existing experimental results shows validation of the minimum required $W/d_{\max}$ ratio of 10 by ASTM for direct shear tests. Rather, almost all of them show that SSE cannot be eliminated by using specimens having a $W/d_{\max}$ ratio of 10. The minimum required $W/d_{\max}$ ratio of 10 stipulated by ASTM is invalidated and needs to be revised.

Table 8. Summarization of the analyses on existing test data obtained to investigate the SSE.

References		Tested $W/d_{\max}$	Minimum Require $W/d_{\max}$	Minimum Tested W/d_{max} Large Enough to Remove SSE	Minimum Required $W/d_{\max}$ of ASTM
Parsons [25]		71, 119, 143	~71	Yes at 71	Unknown
Rathee [26]		10, 48	Unknown	Not at 10	Invalidated
Cerato and Lutenegger [8]		12, 20, 61	Unknown	Not at 20	Invalidated
Wu et al. [30]		95, 285	Unknown	Not at 95	Invalidated
Mirzaeifar et al. [32]		46, 77, 231	~77	Not at 46	Invalidated
Palmeira and Milligan [28]		50, 127, 833	≤50	Yes at 50	Unknown
Ziaie Moayed et al. [34]	Sand	75, 125, 375	Unknown	Not at 75	Invalidated
	Silty sand I	75, 125, 375	Unknown	Not at 75	Invalidated
	Silty sand II	75, 125, 375	~125	Not at 75	Invalidated
	Silty sand III	75, 125, 375	≤75	Yes at 75	Unknown

Table 8 also shows that a $W/d_{\max}$ ratio of 50 is large enough based on some of the existing data while some other results indicate that even a specimen having $W/d_{\max}$ ratio as large as 75 is not large enough to remove SSE. In addition, more experimental results are particularly necessary with specimens having $W/d_{\max}$ ratio between 10 and 50. With fine particle materials, this is only possible with shear box smaller than the minimum required specimen size of 50 mm, stipulated by the ASTM. To fill this gap, a series of direct shear tests were performed on specimens with d_{max} in the range of 0.85 to 6 mm by using a mini shear box of 38 mm × 38 mm × 45 mm, a small shear box of 60 mm × 60 mm × 45 mm and a large shear box of 300 mm × 300 mm × 180 mm. The tested specimens then have a $W/d_{\max}$ ratio between 10 and 353. In this paper, the experimental results of these laboratory tests are first presented. Minimum required $W/d_{\max}$ ratio is identified and proposed to eliminate SSE.

3. Laboratory Tests

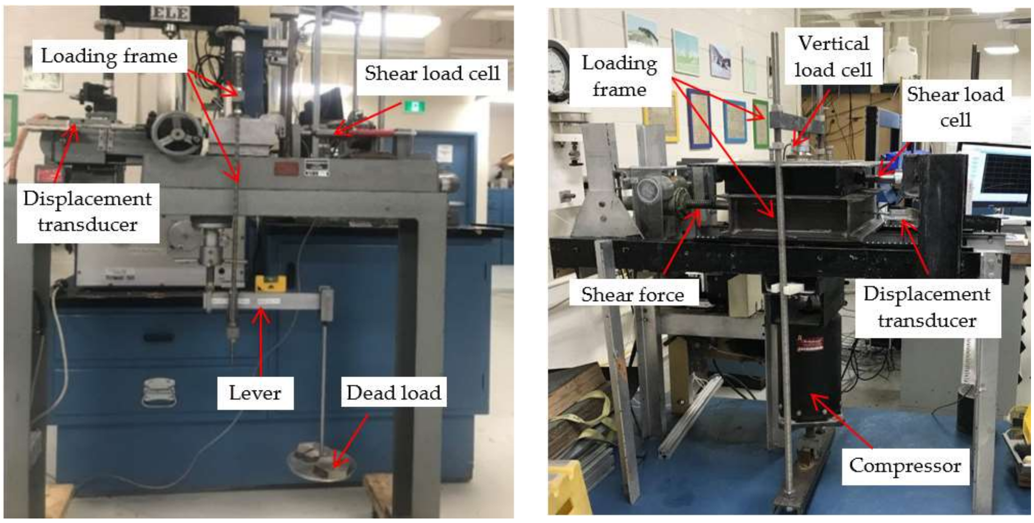
3.1. Test Apparatus

In the Geotechnical Laboratory of Polytechnique Montreal, there are a standard (Figure 1a) and a large (Figure 1b) direct shear test system. The standard direct shear test apparatus is equipped with a small (standard) shear box of 60 mm × 60 mm × 45 mm while the large direct shear apparatus is equipped with a large shear box of 300 mm × 300 mm × 180 mm. A mini shear box of 38 mm × 38 mm × 45 mm was designed and manufactured by the Geotechnical Laboratory of Polytechnique Montreal in order to test the SSE of fine particle materials with a $W/d_{\max}$ ratio smaller than 50 based on the critical review of Deiminiat et al. [76]. Figure 2 shows a picture of the three shear boxes used in this study. The mini shear box was made to have the same external dimensions as those of the small (standard) shear box in order for it to be compatible with the standard direct shear test system. The testing procedure is thus mainly the same as the standard one of ASTM with the small (standard) shear box. The only difference is in the normal stress loading system. The lever system is removed and the normal stress is applied by the self-weight of the loading frame and addition of dead load on the holding plate.

3.2. Materials and Testing Procedure

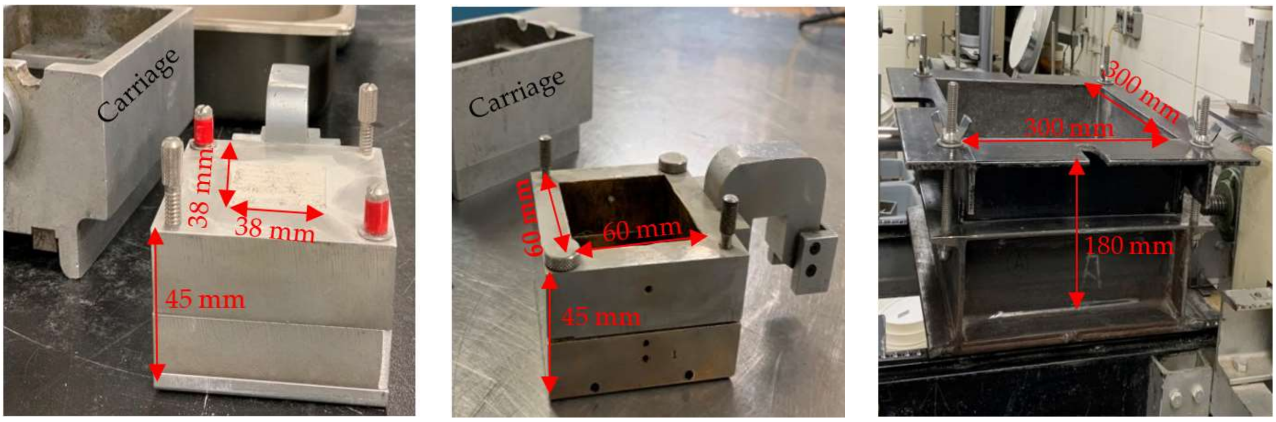
Keeping in mind that the scope of this study is to only analyze the SSE on the peak internal effective friction angle (hereafter called "friction angle" for the sake of simplicity) of granular material, it is very important to make sure that all the specimens prepared with a given material in the mini, small, and large shear boxes have the same compactness (density) and water content (dry).

The testing materials in this study are two types of waste rocks, called WR 1 and WR 2, respectively. These waste rocks contain sub-angular and sub-rounded particles. Figure 3 shows pictures of the original WR 1 (Figure 3a) and WR 2 (Figure 3b).



(**a**) (**b**)

Figure 1. Direct shear apparatuses: (**a**) standard one for mini and small shear boxes; (**b**) large one for large shear box.



(a) (b) (c)

Figure 2. Shear boxes used in this study: (**a**) house-made mini square shear box (38 mm × 38 mm × 45 mm); (**b**) small (standard) square shear box (60 mm × 60 mm × 45 mm); (**c**) large square shear boxes (300 mm × 300 mm × 180 mm).

(a) (b)

Figure 3. Photos of original (**a**) WR 1 and (**b**) WR 2.

The two types of waste rocks were first sieved in different portions having particle sizes ranging from 0.08–0.85 mm, 0.85–1.19 mm, 1.19–1.4 mm, 1.4–2.36 mm, 2.36–3.36 mm, 3.36–5 mm, and 5–6 mm. Mixtures were made to obtain dry samples having a $d_{\max}$ of 0.85, 1.19, 1.4, 2.36, 3.36, 5, and 6 mm. Details of the different portions used to make the different mixtures are given in Table 9.

Table 9. Portions of waste rocks used to make the different mixtures.

Ranges of Particle Sizes	Masse of Different Portions (g)						
	d_{max} = 6 mm	5 mm	3.36 mm	2.36 mm	1.4 mm	1.19 mm	0.85 mm
5 to 6 mm	3618						
3.36 to 5 mm	5899	6190					
2.36 to 3.36 mm	4314	4432	5062				
1.4 to 2.36 mm	3272	3227	2586	4663			
1.19 to 1.4 mm	524	721	789	1334	2301		
0.85 to 1.19 mm	1326	1379	1603	2202	2220	3283	
0.63 to 0.85 mm	121	125	369	327	1549	2401	6500
0.315 to 0.63 mm	370	385	291	449	710	853	1440
0.16 to 0.315 mm	345	359	408	315	539	568	590
0.08 to 0.16 mm	1063	1105	791	972	771	832	1300
≤0.08 mm	2942	3060	2531	2691	2186	2324	3550

Figure 4 shows the target and obtained grain size distribution curves of samples having different d_{max}. The grain size distribution curves were also used to produce seven other materials by WR2. There are thus 14 samples prepared for the direct shear tests with the three different shear boxes. It should also be mentioned that the scope of this paper is to study the SSE, not the scaling-down techniques or the effect of d_{max} on the friction angle of granular materials. The different d_{max} values are thus used here as an identification of one material. Each d_{max} along with the type of waste rocks constitute a distinct material.

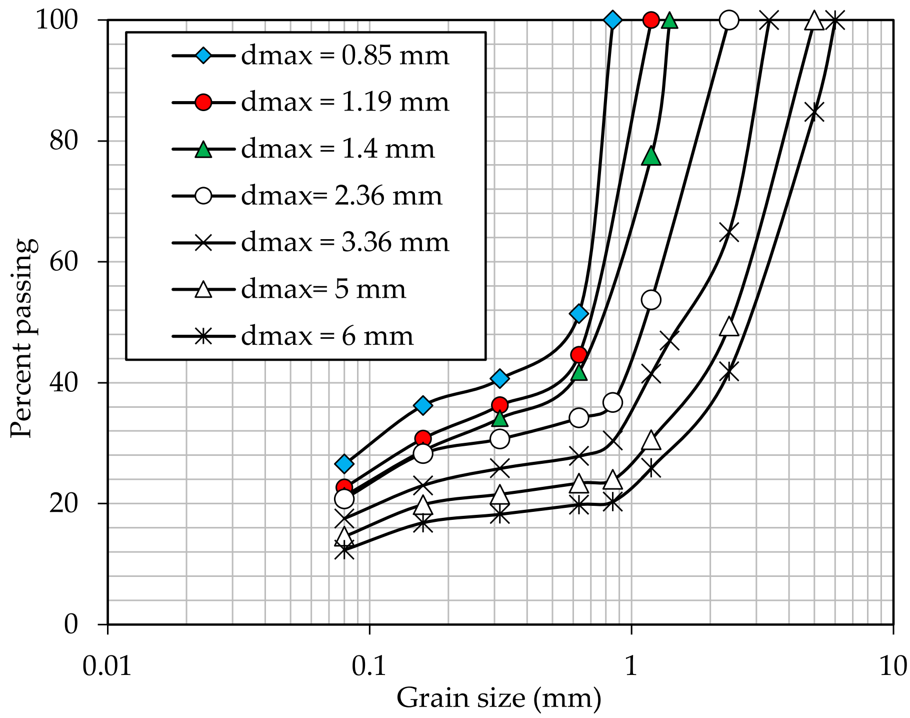


Figure 4. Grain size distribution curves of the samples with different d_{max} (same for WR 1 and WR 2).

All the specimens for the mini, small, and large shear boxes were prepared with dry waste rocks in the loosest state in order to ensure that the variation of the test results is only associated with the variation of specimen sizes; the effects of other influencing factors (density, water content, particles shapes, etc.) on the test results are excluded. The specimen was prepared with a spoon by slowly placing waste rocks into each shear box without any compaction. The required mass of material was calculated according to the box volume and the desired dry density of 1450 kg/m^3. After filling the box to the top edge of the upper box, the surface of the specimen was smoothed with a brush and the rigid plate was placed slowly to avoid any shock or compaction of the material.

Table 10 shows the shear boxes and specimen sizes to d_{max} ratios along with the maximum void ratios (e_{max}) for the two types of waste rocks. The values of e_{max} were estimated by following ASTM C29/C29M-17a [77]. One notes a decrease of e_{max} with an increased d_{max} value. This is straightforward to understand. As seen in Figure 4, the materials with larger d_{max} have better gradations. With a well graded material, the pore or void spaces can easily be filled with fine particles, resulting in a denser material compared to a poorly graded material. This, along with the dense and heavy large particles associated with a large d_{max} value, results in a small e_{max}. However, it should be noted that the scope of this study is to evaluate the SSE of direct shear tests. The value of d_{max} is only an identification of the material. Focus should be put on the variation of friction angle as function of specimen width for a given material (defined by the type of material and a d_{max} value), not on the variation of physical or mechanical properties as function of d_{max} value.

Table 10. Shear boxes and specimen size to d_{max} ratios, used in the direct shear tests on the two types of waste rocks.

d_{max} (mm)	e_{max}		38 mm × 38 mm × 45 mm		60 mm × 60 mm × 45 mm		300 mm × 300 mm × 180 mm	
	WR 1	**WR 2**	W/d_{max}	T/d_{max}	W/d_{max}	T/d_{max}	W/d_{max}	T/d_{max}
0.85	0.93	0.84	45	53	71	53	353	212
1.19	0.87	0.79	32	38	50	38	252	151
1.4	0.84	0.73	27	32	43	32	214	129
2.36	0.88	0.75	16	19	25	19	127	76
3.36	0.85	0.74	11	13	18	13	89	54
5.0	0.80	0.77	–	–	12	9	60	36
6.0	0.78	0.72	–	–	10	8	50	30

From Table 10, one sees that there are no tests planned with the mini box for the specimens with d_{max} of 5 and 6 mm in order to respect the minimum required specimen size to d_{max} ratio of 10 defined by ASTM. All the 19 specimen sizes meet the requirement of ASTM with W/d_{max} ranging from 10 to 353. Since each d_{max} along with the type of waste rocks is only used as an identification of a material, the testing program contains 14 materials (made of WR 1 and WR 2) completely different from each other.

As the shear strength of each specimen is obtained by direct shear tests with three normal stresses (50, 100, and 150 kPa), 57 direct shear tests were carried out for each type of waste rocks. A total number of 114 direct shear tests were realized for the two types of waste rocks. The direct shear tests were conducted by applying constant rates of 0.015 mm/s (0.9 mm/min) and 0.025 mm/s (1.5 mm/min) for the standard and large shear apparatuses, respectively.

3.3. Test Results

Figure 5 shows some typical shear stress and shear displacement curves obtained by using the mini (Figure 5a), small (Figure 5b) and large (Figure 5c) shear boxes on WR 1 with a d_{max} value of 0.85 mm. It can be seen that material tested with different boxes at a given normal stress has the same mechanical behavior. For example, at a normal stress of 50 kPa, the material tested with the mini, small, and large boxes has a mechanical behavior of loose sand. At a normal stress of 150 kPa, the material tested with the three boxes starts to have a mechanical behavior of dense sand. These results indirectly indicate that the specimens prepared in the three different boxes have the same state and density. Subsequently, they have the same mechanical behavior under a given normal stress: loose sand under a normal stress of 50 kPa and slightly dense sand under a normal stress of 150 kPa.

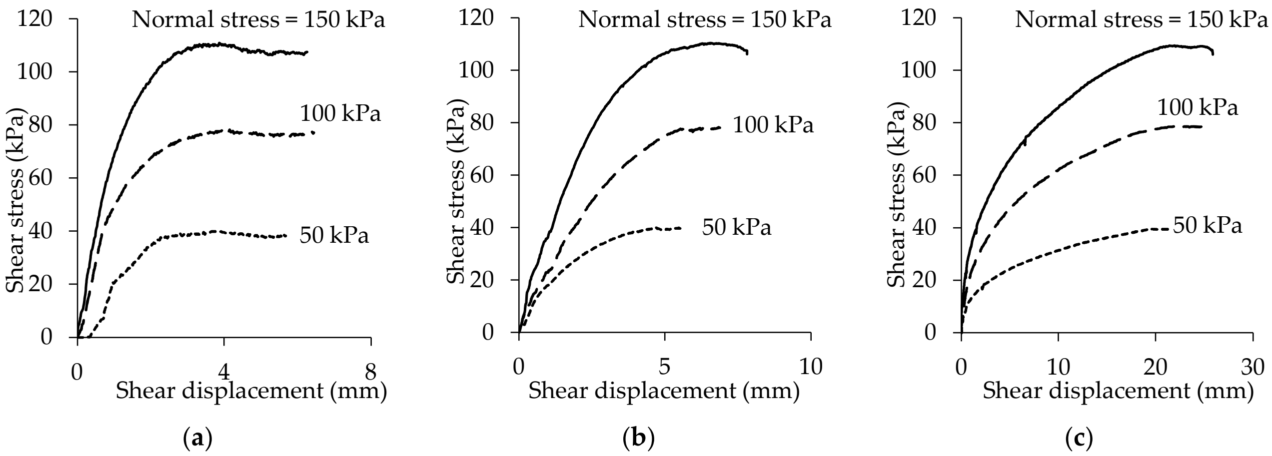


Figure 5. Shear stress vs. shear displacement curves of WR 1 specimens with d_{max} value of 0.85 mm under three normal stresses obtained with the mini (**a**), small (**b**) and large (**c**) shear boxes.

By taking the peak value of shear stress for each curve, one then obtains three shear stresses at yield, each corresponding to one normal stress. The friction angle can then be obtained by applying the linear fitting technique without imposing the straight line passing to the origin in Mohr plane.

Table 11 shows the obtained friction angles of all the specimens with different d_{max} for WR 1 and WR 2. Once again, each d_{max} along with the type of waste rocks can only be considered as an identification of a material. The table thus presents the friction angles of 38 samples made of the 14 materials. It is unnecessary to analyze how the friction angles change with the value of d_{max} or with the type of waste rocks.

Table 11. Friction angles of the specimens obtained by extra small, small, and large shear boxes for WR 1 and WR 2.

d_{max} (mm)	38 mm × 38 mm × 45 mm				60 mm × 60 mm × 45 mm				300 mm × 300 mm × 180 mm			
	W/d_{max}	T/d_{max}	ϕ (°)		W/d_{max}	T/d_{max}	ϕ (°)		W/d_{max}	T/d_{max}	ϕ (°)	
			WR 1	WR 2			WR 1	WR 2			WR 1	WR 2
0.85	45	53	37.1	35.3	71	53	37	35.2	353	212	36.9	35.0
1.19	32	38	38	36.2	50	38	37.9	36.1	252	151	37.5	36.0
1.4	27	32	38.7	37.2	43	32	38.0	36.4	214	129	37.7	36.2
2.36	16	19	40.9	38.2	25	19	39.1	37.3	127	76	37.9	37.1
3.36	11	13	42.1	40.5	18	13	40.2	39.3	89	54	38.7	37.4
5	–	–	–	–	12	9	41.4	40.1	60	36	39.5	38.4
6	–	–	–	–	10	8	43.0	40.9	50	30	39.9	39.2

Figure 6 shows the variations of friction angles as function of W/d_{max} for the specimens made of WR 1 (Figure 6a) and WR 2 (Figure 6b), respectively. From Figure 6a, it is seen that the friction angle of the specimens with d_{max} = 0.85 mm remains constant as the W/d_{max} ratio increases from 45 to 353. For the specimens with d_{max} = 1.19 mm, the friction angle decreases by 0.1° as W/d_{max} increases from 32 to 50 and remains almost constant as W/d_{max} further increases to 252. For the specimens with d_{max} = 1.4 mm, the friction angle decreases by 0.7° when W/d_{max} increases from 27 to 43 and it remains almost constant when W/d_{max} further increases from 43 to 214. These results tend to indicate that a W/d_{max} ratio of 43 to 50 is large enough to remove the SSE while the minimum required W/d_{max} ratio of 10 suggested by the ASTM is too small for eliminating the SSE.

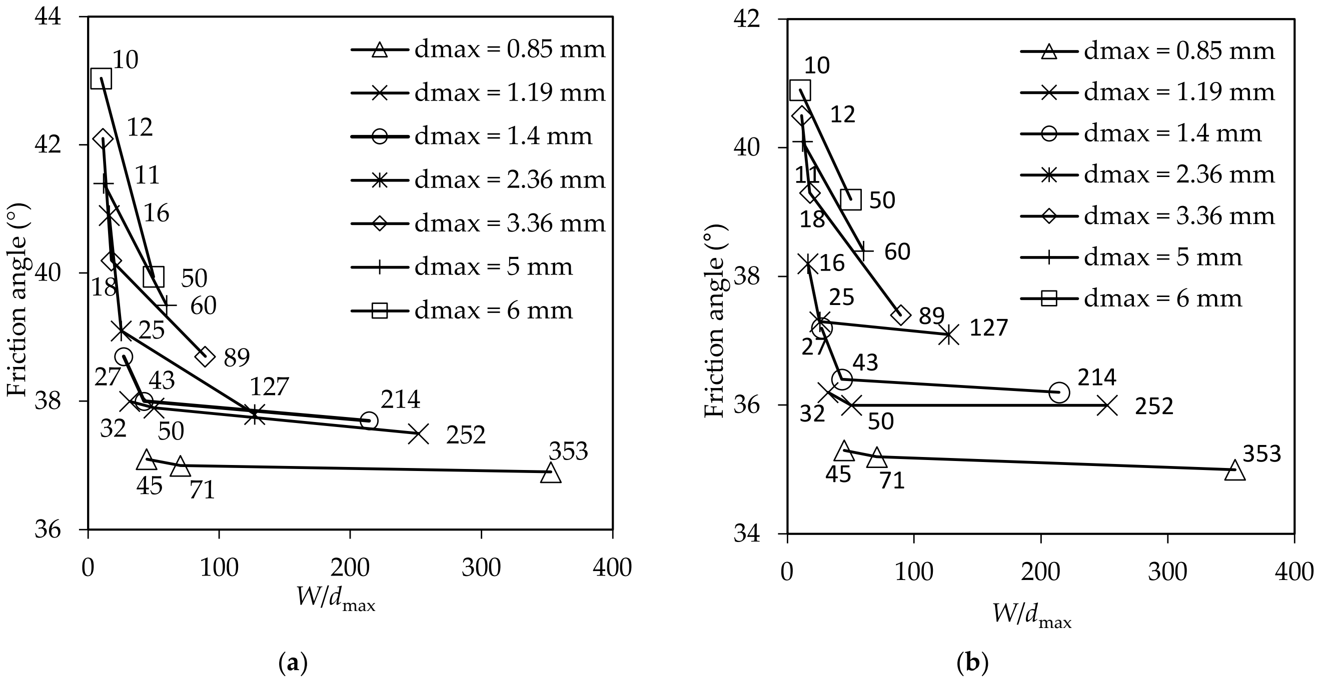


Figure 6. Variations of the friction angles of the specimens as a function of W/d_{max} for the specimens made of (**a**) WR 1 and (**b**) WR 2.

For the specimens with d_{max} = 2.36 mm, the friction angle decreases by 1.8° as W/d_{max} increases from 16 to 25 and then decreases by 1.2° when W/d_{max} further increases from 25 to 127. Similarly for the specimens with d_{max} = 3.36 mm, the friction angle decreases by at least 1.9° as W/d_{max} increases from 11 to 18 and decreases by 1.5° as W/d_{max} further increases from 18 to 89. These test results indicate that a W/d_{max} ratio of 18 or 25 is not large enough to eliminate the SSE. The minimum required W/d_{max} ratio of 10 defined by ASTM is invalidated again.

For the specimens with d_{max} = 5 mm, the friction angle decreases by 2.2° as W/d_{max} increases from 12 to 60. For the specimens with d_{max} = 6 mm, the friction angle decreases by 3.1° as W/d_{max} increases from 10 to 50. These results show once again that the minimum required W/d_{max} ratio of 10 defined by ASTM is not large enough to remove SSE.

All the above test results confirm what has been illustrated by Deiminiat et al. [76]. The minimum required W/d_{max} ratio of 10 defined in the ASTM is not validated for the fine particle material with a d_{max} value of 0.85 mm due to the lack of test results with W/d_{max} ratio in the range of 10 to 45, but is clearly invalidated for the coarse particle materials with d_{max} ranging from 3.36 to 6 mm. A W/d_{max} ratio of 10 is not large enough to eliminate the SSE on the friction angles of granular materials. The minimum required W/d_{max} ratio of ASTM should be revised.

Similar observations can be made on the experimental results shown in Figure 6b for WR 2. The friction angle of the specimens with d_{max} = 0.85 mm remains constant as W/d_{max} increases from 45 to 353. For the specimens with d_{max} = 1.19 mm, the friction angle decreases by 0.2° as W/d_{max} increases from 32 to 50 and remains almost constant as W/d_{max} further increases to 252. The friction angle of the specimens with d_{max} = 1.4 mm decreases by 0.8° when W/d_{max} increases from 27 to 43 and remains almost constant when W/d_{max} further increases from 43 to 214. For the specimens with d_{max} = 2.36 mm, the friction angle decreases by 0.9° as W/d_{max} increases from 16 to 25 and then decreases by 0.2° when W/d_{max} further increases from 25 to 127. The friction angle of the specimens with d_{max} = 3.36 mm decreases by at least 1.2° as W/d_{max} increases from 11 to 18 and decreases by 1.9° as W/d_{max} further increases from 18 to 89. For the specimens with d_{max} = 5 mm, the friction angle decreases by 1.7° as W/d_{max} increases from 12 to 60. The friction angle of the specimens with d_{max} = 6 mm decreases by 1.7° as W/d_{max} increases from 10 to 50. These results show once again that a W/d_{max} ratio of 43 to 50 is large enough to eliminate the SSE while a W/d_{max} ratio of 10 to 32 is too small to eliminate the SSE on the friction angles of granular materials. The minimum required W/d_{max} ratio of 10 defined in the ASTM needs to be revised [76].

4. Identification of the Minimum Required W/d_{max} Ratio to Eliminate SSE

With the experimental results of fine particle materials with d_{max} values ranging from 0.83 to 1.4 mm shown in Figure 6, a W/d_{max} ratio of 43 to 50 seems to be large enough to eliminate the SSE on the friction angles of granular materials. It still remains unclear if these ratios can be applied to the specimens with d_{max} values ranging from 2.36 to 6 mm to remove the SSE.

One recalls that each d_{max} along with the type of waste rocks must be considered as the identification of one material. It is thus normal to see the test result points as a cloud, as shown in Figure 6. It is however very difficult, with such presentation, to identify a unique value respectively for the minimum required W/d_{max} ratio that can be considered as large enough to eliminate the SSE on the friction angle of granular materials.

Table 12 shows the friction angles of experimental results obtained in this study and those of existing data, normalized by the measured friction angle of sample having a large enough W/d_{max} ratio. Only a selected part of the existing data presented in Table 8 is included and presented in the table because the normalization can only be made on the experimental results with at least one specimen having W/d_{max} ratio large enough to eliminate the SSE. The experimental results of Rathee [26] and Wu et al. [30] are thus excluded.

Table 12. Normalized friction angles of the experimental results obtained in this study and taken from the literature.

Id. of Material	W/d_{max}	T/d_{max}	ϕ (°)	Normalized ϕ	Id. of Material	W/d_{max}	T/d_{max}	ϕ (°)	Normalized ϕ	References
WR 1, d_{max} = 0.85 mm	45 71 353	53 53 212	37.1 37.0 36.9	1.005 1.003 1	WR 2, d_{max} = 0.85 mm	45 71 353	53 53 212	35.3 35.2 35.0	1.009 1.006 1	This study
WR 1, d_{max} = 1.19 mm	32 50 252	38 38 151	38.0 37.9 37.5	1.013 1.011 1	WR 2, d_{max} = 1.19 mm	32 50 252	38 38 151	36.2 36.1 36.0	1.006 1.002 1	This study
WR 1, d_{max} = 1.4 mm	27 43 214	32 32 129	38.7 38.0 37.7	1.027 1.008 1	WR 2, d_{max} = 1.4 mm	27 43 214	32 32 129	37.2 36.4 36.2	1.028 1.006 1	This study
WR 1, d_{max} = 2.36 mm	16 25 127	19 19 76	40.9 39.1 37.8	1.082 1.034 1	WR 2, d_{max} = 2.36 mm	16 25 127	19 19 76	38.2 37.3 37.1	1.030 1.005 1	This study
WR 1, d_{max} = 3.36 mm	11 18 89	13 13 54	42.1 40.2 38.7	1.088 1.039 1	WR 2, d_{max} = 3.36 mm	11 18 89	13 13 54	40.5 39.3 37.4	1.083 1.051 1	This study
WR 1, d_{max} = 5 mm	12 60	9 36	41.4 39.5	1.048 1	WR 2, d_{max} = 5 mm	12 60	9 36	40.1 38.4	1.044 1	This study
Gravel, d_{max} = 5 mm, Dr = 25%	12 20 61	5 8 36	42.0 36.5 34.0	1.235 1.074 1	Gravel, d_{max} = 5 mm, Dr = 55%	12 20 61	5 8 36	44.5 41.0 40.2	1.107 1.020 1	[8]
Gravel, d_{max} = 5 mm, Dr = 85%	12 20 61	5 8 36	45.5 43.0 42.0	1.083 1.024 1						[8]
Sand, d_{max} = 1.2 mm	50 833	27 833	50.1 49.4	1.014 1						[28]

Regarding the direct shear tests of Palmeira and Milligan [28], the results obtained with the rectangular shear box should be excluded in order to avoid any specimen shape effect. The remaining results should be included because it is difficult to justify that a specimen of 1 m by 1 m is still not large enough to eliminate the SSE with a tested material having a $d_{\max}$ = 1.2 mm and a $W/d_{\max}$ ratio of 833.

In this study, the experimental results of Ziaie Moayed [34] have also been excluded because their experimental results involve too many uncertainties pertaining to the SSE of friction angles. For example, their tested materials were prepared at optimum water contents. The very high apparent cohesions obtained with the small shear box can be well explained by the suction associated with the unsaturated state of the sample. However, the very small and even negative apparent cohesion obtained by the medium and large shear boxes cannot be explained by the unsaturated state of the samples. In addition, the shear stress–shear displacement curves clearly showed that the specimens in the small box received more compactness, having a mechanical behavior of dense sand while the specimens in the large box did not receive enough compactness, showing a mechanical behavior of loose sand. The problem of compactness can further be confirmed by the number of layers during their specimen preparation: three layers both in the small and medium sizes of shear box, five layers in the large shear box [34].

Regarding the experimental results of Mirzaeifar et al. [32], it is noted that the desired densities of 1.5, 1.58, and 1.67 g/cm^3 were obtained by compacting the sand in 4, 6 and 8 layers, respectively. With the same number of layers in small, medium and large shear boxes, the resulting densities could be expected very different. Their test results thus not only involve the SSE, but also the effects of compactness or density. The experimental results of Mirzaeifar et al. [32] should thus also be excluded in the identification of the minimum required $W/d_{\max}$ ratio.

Figure 7 presents the variation of the normalized friction angles as a function of $W/d_{\max}$ of the experimental results obtained in this study and taken from the literature. It can be seen that the normalized friction angle varies from 1.12 to 1 depending on the W/d_{max} ratio. The normalized friction angle decreases as $W/d_{\max}$ increases from 10 to a certain value before it becomes constant when the $W/d_{\max}$ ratio further increases from this critical value to a value as high as 353. The critical value of $W/d_{\max}$ ratio beyond which the normalized friction remains constant is the searched minimum required $W/d_{\max}$ ratio to eliminate the SSE on friction angle of granular materials.

To identify these critical values, one first draws a horizontal line at normalized friction angles equal to 1. Eye-based best-fitted curves are then plotted to the experimental results having the $W/d_{\max}$ ratio between 10 and 50. One sees that the critical $W/d_{\max}$ ratio should be in the range of 50 to 70. One recommends a value of 60 for the minimum required $W/d_{\max}$ ratio, identified as large enough to eliminate the SSE. This value is chosen as a compromise between accuracy and practical convenience.

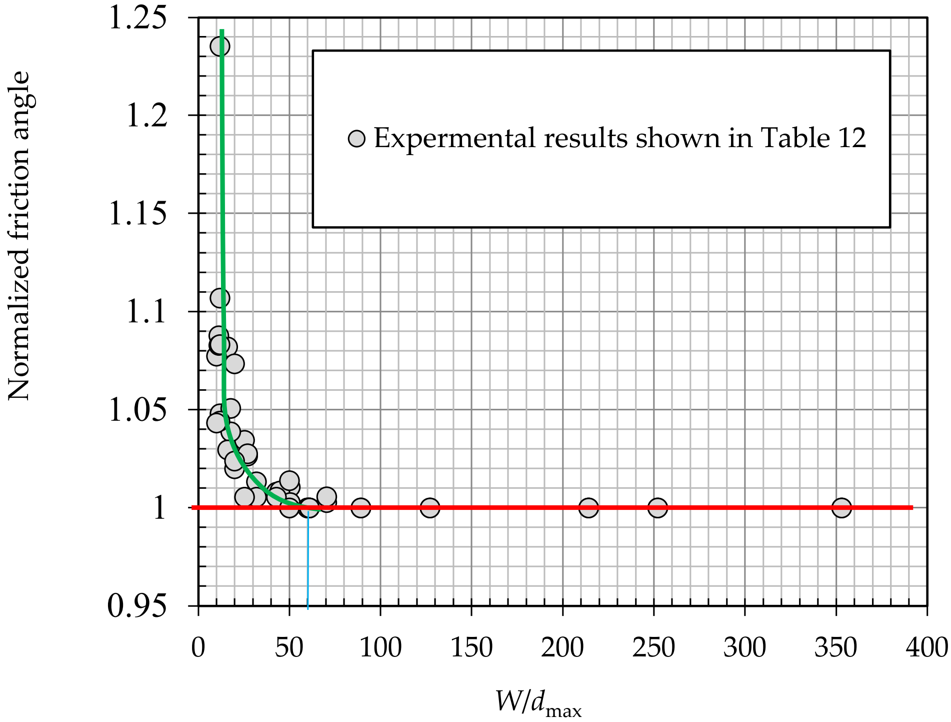


Figure 7. Variations of the normalized friction angles as a function of $W/d_{\max}$ of the experimental results obtained in this study and taken from the literature. The minimum required specimen size is identified as $W/d_{\max}$ = 60.

5. Discussions

The direct shear test is a very old but still regularly used method to determine the shear strength of geomaterials [1–7]. Unlike triaxial compression tests, a direct shear test imposes a sliding plane determined by upper and lower half boxes. Several shortages can be attributed to this imposed sliding plane. By following the standard of ASTM D3080 published in 1972, one can doubtlessly ensure higher quality of experimental results. Nevertheless, people are aware of the limitations. ASTM D3080 has to be regularly revised and updated every eight years. Recently, ASTM D3080/D3080M-11 [42] was temporarily withdrawn by the ASTM technical committees due to there being an excess of eight years since the last update. It seems that main concern of this update is related to the gap thickness to be left. One can expect that the updated version remains unchanged with respect to the minimum required specimen sizes. The current study is thus useful to inform its future updates.

This experimental study leads to a recommendation of a minimum required W/d_{max} ratio of 60. It is interesting to note that the same value was recommended by Wang and Gutierrez [39], who studied the SSE of direct shear tests through 2D discrete element modeling and made such a recommendation based on their experience (personal communication by email between the first author and Prof. Jianfeng Jerry Wang on 25 October 2021).

Despite the important and interesting discovery of this study, one has to point out that this experimental study involves several limitations.

In this study, the effect of specimen width was evaluated by trying to keep other influencing parameters constant. For one given material, all the shear tests with different shear boxes were performed with the same compact state and moisture content. However, due to a lack of shear boxes, the thickness of tested specimens meets the minimum required $T/d_{\max}$ ratio of ASTM, but is not constant. The influence of this parameter, the gap thickness between upper and lower half boxes as well as the specimen aspect ratio W/T were not taken into account [39,78]. More experimental work is thus necessary not only to verify the validity of the recommended value of the minimum required $W/d_{\max}$ ratio, but also to verify if the minimum required $T/d_{\max}$ ratio of ASTM is large enough to eliminate the SSE in direct shear tests [39].

Another limitation of this work is associated with the unique consideration of specimen sizes to $d_{\max}$ ratios. It is, however, well-known that the mechanical properties of geomaterials are not only controlled by the size of $d_{\max}$, but also by the content of $d_{\max}$. Two materials having the same $d_{\max}$ with different contents can behave differently. It is thus interesting and important to take into account other influencing factors, such as the portion of the coarsest particles, median size d_{50}, coefficient of uniformity, particle shapes, contents of fine and coarse particles, and crushability of particles. It is also well-known that the friction angle of geomaterials depends on the compactness, water content, and confining pressure. More experimental work is necessary with more types of materials of different sources by taking into account these different influencing factors.

Finally, more experimental work can be necessary to evaluate if the minimum required specimen diameter over $d_{\max}$ ratio of 6 specified by ASTM D4767 [79] is large enough to eliminate SSE of triaxial compression tests by following the methodology presented in this study.

6. Conclusions

The minimum required specimen width to d_{max} ratio of ASTM has been revised based on an analysis on existing data and new experimental results. The following conclusions can be drawn:

- The experimental results confirm what has been reported in [76], who showed that the minimum requirement of ASTM was not validated for fine particle materials due to the lack of experimental data with W/d_{max} ranging from 10 to 50, but invalidated for coarse particle materials. An update is necessary for the minimum required ratio between specimen sizes and d_{max}, stipulated by the ASTM D3080/D3080M [42] for direct shear tests.
- The minimum required W/d_{max} ratio to eliminate the SSE on the shear strengths of granular materials is identified as equal to 60.
- For fine particle materials having a d_{max} not larger than 1 mm, using the standard shear boxes having W = 60 mm automatically results in $W/d_{max} \geq 60$. The obtained friction angles can be considered as fully representative to that of the tested material in field conditions. The ASTM D3080/D3080M-11 can thus continue to be applied without any problem of SSE.
- For granular materials having d_{max} larger than 1 mm, applying the minimum requirements of ASTM D3080/D3080M-11 may result in a W/d_{max} ratio much smaller than the identified minimum required W/d_{max} ratio. The obtained friction angles can be erroneous.

More experimental works are necessary, not only with shear boxes of different sizes, but also with more materials by considering different testing conditions, including different densities, water contents, particle shapes, normal stresses, etc.

Author Contributions: Conceptualization, L.L. and A.D.; methodology, A.D. and L.L.; formal analysis, A.D., L.L. and F.Z.; investigation, A.D., F.Z. and L.L.; writing—original draft preparation, A.D. and F.Z.; writing—review and editing, L.L., A.D. and F.Z.; supervision, L.L.; project administration, L.L.; funding acquisition, L.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by Natural Sciences and Engineering Research Council of Canada (NSERC RGPIN-2018-06902), Fonds de recherche du Québec—Nature et Technologies (FRQNT 2017-MI-202860), and Mitacs (IT12569).

Data Availability Statement: Not applicable.

Acknowledgments: The authors acknowledge the financial support from the Natural Sciences and Engineering Research Council of Canada (NSERC RGPIN-2018-06902), Fonds de recherche du Québec—Nature et Technologies (FRQNT 2017-MI-202860), industrial partners of the Research Institute on Mines and the Environment (RIME UQAT-Polytechnique; http://rime-irme.ca/, accessed on 15 January 2022), and Mitacs Elevate Postdoctoral Fellowship (IT12569). Samuel Chenier, Eric Turgeon and Noura El-Harrak are gratefully acknowledged for their assistance in the laboratory work.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Goodrich, E.P. Lateral earth pressure and related phenomena. *Trans. Am. Soc. Civil Eng.* **1904**, *53*, 272–304. [CrossRef]
2. Casagrande, A.; Albert, S.G. *Research on the Shearing Resistance of Soils*; Massachusetts Institute of Technology: Cambridge, MA, USA, 1932.
3. Casagrande, A. Characteristics of cohesionless soils affecting the stability of slopes and earth fills. *J. Boston Soc. Civ. Eng.* **1936**, *23*, 13–32.
4. Terzaghi, K. The shearing resistance of saturated soils and the angle between the planes of shear. In Proceedings of the 1st International Conference on Soil Mechanics and Foundation Engineering, Harvard, Boston, MA, USA, 22–26 June 1936; Volume 2, pp. 54–56.
5. Cooling, L.F.; Smith, D.B. The shearing resistance of soils. In Proceedings of the 1st International Conference on Soil Mechanics and Foundation Engineering, Harvard, Boston, MA, USA, 22–26 June 1936; pp. 37–41. [CrossRef]
6. Terzaghi, K.; Peck, R.B. *Soil Mechanics in Engineering Practice*; Wiley: New York, NY, USA, 1948.
7. Hutchinson, J.N.; Rolfsen, E.N. Large scale field shear box test and quick clay. *Adv. Civ. Eng.* **1962**, *28*, 31–42.
8. Cerato, A.B.; Lutenegger, A.J. Specimen size and scale effects of direct shear box tests of sands. *Geotech. Test. J.* **2006**, *29*, 1–10.
9. Afzali-Nejad, A.; Lashkari, A.; Shourijeh, P.T. Influence of particle shape on the shear strength and dilation of sand-woven geotextile interfaces. *Geotext. Geomembr.* **2017**, *45*, 54–66. [CrossRef]
10. Afzali-Nejad, A.; Lashkari, A.; Farhadi, B. Role of soil inherent anisotropy in peak friction and maximum dilation angles of four sand-geosynthetic interfaces. *Geotext. Geomembr.* **2018**, *46*, 869–881. [CrossRef]
11. Zhang, Z.; Sheng, Q.; Fu, X.; Zhou, Y.; Huang, J.; Du, Y. An approach to predicting the shear strength of soil-rock mixture based on rock block proportion. *Bull. Eng. Geol. Environ.* **2019**, *79*, 2423–2437. [CrossRef]
12. Cai, H.; Wei, R.; Xiao, J.Z.; Wang, Z.W.; Yan, J.; Wu, S.F.; Sun, L.M. Direct shear test on coarse gap-graded fill: Plate opening size and its effect on measured shear strength. *Adv. Civ. Eng.* **2020**, 5750438. [CrossRef]
13. Zahran, K.; Naggar, H.E. Effect of sample size on TDA shear strength parameters in direct shear tests. *Transp. Res. Rec.* **2020**, *2674*, 1110–1119. [CrossRef]
14. Motahari Tabari, S.; Shooshpasha, I. Evaluation of coarse-grained mechanical properties using small direct shear test. *Int. J. Geotech. Eng.* **2021**, *15*, 667–679. [CrossRef]
15. Xue, Z.F.; Cheng, W.C.; Wang, L.; Song, G. Improvement of the shearing behaviour of loess using recycled straw fiber reinforcement. *KSCE J. Civ. Eng.* **2021**, 1–17. [CrossRef]
16. Deiminiat, A.; Li, L. Experimental study on the reliability of scaling down techniques used in direct shear tests to determine the shear strength of rockfill and waste rocks. *Civil Eng.* **2022**, *3*, 35–50. [CrossRef]
17. Jewell, R.A.; Wroth, C.P. Direct shear tests on reinforced sand. *Géotechnique* **1987**, *37*, 53–68. [CrossRef]
18. Jewell, R.A. Direct shear test on sand. *Géotechnique* **1989**, *39*, 309–322. [CrossRef]
19. Shibuya, S.; Mitachi, T.; Tamate, S. Interpretation of direct shear box testing of sands as quasi-simple shear. *Géotechnique* **1997**, *47*, 769–790. [CrossRef]
20. Lings, M.L.; Dietz, M.S. An improved direct shear apparatus for sand. *Géotechnique* **2004**, *54*, 245–256. [CrossRef]
21. Amirpour Harehdasht, S.; Karray, M.; Hussien, M.N.; Roubtsova, V.; Chekired, M. Influence of particle size and gradation on the stress-dilatancy behavior of granular materials during drained triaxial compression. *Int. J. Geomech.* **2017**, *17*, 1–20. [CrossRef]
22. Drugan, W.J.; Willis, J.R. A micromechanics-based nonlocal constitutive equation and estimates of representative volume element size for elastic composites. *J. Mech. Phys. Solids* **1996**, *44*, 497–524. [CrossRef]
23. Kanit, T.; Forest, S.; Galliet, I.; Mounoury, V.; Jeulin, D. Determination of the size of the representative volume element for random composites: Statistical and numerical approach. *Int. J. Solids Struct.* **2003**, *40*, 3647–3679. [CrossRef]
24. Wen, R.; Tan, C.; Wu, Y.; Wang, C. Grain size effect on the mechanical behavior of cohesionless coarse-grained soils with the discrete element method. *Adv. Civ. Eng.* **2018**, *2018*, 4608930. [CrossRef]
25. Parsons, J.D. Progress report on an investigation of the shearing resistance of cohesionless soils. In Proceedings of the 1st International Conference on Soil Mechanics and Foundation Engineering, Harvard, Boston, MA, USA, 22–26 June 1936; Volume 2, pp. 133–138.
26. Rathee, R.K. Shear strength of granular soils and its prediction by modeling techniques. *J. Inst. Eng.* **1981**, *62*, 64–70.
27. Vucetic, M.; Lacasse, S. Specimen size effect in simple shear test. *ASCE J. Geotech. Eng. Div.* **1982**, *108*, 1567–1585. [CrossRef]
28. Palmeira, E.M.; Milligan, G.W.E. Scale effects in direct shear tests on sand. In Proceedings of the 12th International Conference on Soil Mechanics and Foundation Engineering, Rio De Janeiro, Brazil, 13–18 August 1989; Volume 1, pp. 739–742.
29. Stone, K.J.; Wood, D.M. Effects of dilatancy and particle size observed in model tests on sand. *Soils Found.* **1992**, *32*, 43–57. [CrossRef]

30. Wu, P.-K.; Matsushima, K.; Tatsuoka, F. Effects of specimen size and some other factors on the strength and deformation of granular soil in direct shear tests. *Geotech. Test. J.* **2008**, *31*, 1–20.
31. Alonso, E.E.; Tapias, M.; Gili, J. Scale effects in rockfill behavior. *Géotech. Lett.* **2012**, *2*, 155–160. [CrossRef]
32. Mirzaeifar, H.; Abouzar, A.; Abdi, M.R. Effects of direct shear box dimensions on shear strength parameters of geogrid-reinforced sand. In Proceedings of the 66th Canadian Geotechnical Conference and the 11th Joint CGS/IAH-CNC Groundwater Conference, GeoMontreal, Montrea, QC, Canada, 29 September–3 October 2013; pp. 1–6.
33. Amirpour Harehdasht, S.; Hussien, M.N.; Karray, M.; Roubtsova, V.; Chekired, M. Influence of particle size and gradation on shear strength–dilation relation of granular materials. *Can. Geotech. J.* **2019**, *56*, 208–227. [CrossRef]
34. Ziaie Moayed, R.; Alibolandi, M.; Alizadeh, A. Specimen size effects on direct shear test of silty sands. *Int. J. Geotech. Eng.* **2017**, *11*, 198–205. [CrossRef]
35. Potts, D.M.; Dounias, G.T.; Vaughan, P.R. Finite element analysis of the direct shear box test. *Géotechnique* **1987**, *37*, 11–23. [CrossRef]
36. Wang, J.; Dove, J.E.; Gutierrez, M.S. Discrete continuum analysis of shear band in the direct shear test. *Géotechnique* **2007**, *57*, 513–526. [CrossRef]
37. Zhang, L.; Thornton, C. A numerical examination of the direct shear test. *Géotechnique* **2007**, *57*, 343–354. [CrossRef]
38. Jacobson, D.E.; Valdes, J.R.; Evans, T.M. A numerical view into direct shear specimen size effects. *Geotech. Test. J.* **2007**, *30*, 512–516.
39. Wang, J.; Gutierrez, M. Discrete element simulations of direct shear specimen scale effects. *Géotechnique* **2010**, *60*, 395–409. [CrossRef]
40. DeJong, J.T.; Westgate, Z.J. Role of initial state, material properties, and confinement condition on local and global soil-structure interface behavior. *J. Geotech. Geoenviron. Eng.* **2009**, *135*, 1646–1660. [CrossRef]
41. Lashkari, A.; Jamali, V. Global and local sand–geosynthetic interface behaviour. *Géotechnique* **2021**, *71*, 346–367. [CrossRef]
42. *ASTM D3080/D3080M-11*; Standard Test Method for Direct Shear Test of Soils under Consolidated Drained Conditions (withdrawn 2020). ASTM International: West Conshohocken, PA, USA, 2011.
43. *Eurocode 7-07*; Geotechnical Design; Part 1, General rules: EN 1997-1. The European Union Per Regulation: Brussels, Belgium, 2007.
44. *AS 1289.6.2.2-98*; Soil Strength and Consolidation Tests-Determination of the Shear Strength of a Soil-Direct Shear Test Using a Shear Box. Standards Australia: Sydney, NSW, Australia, 1998.
45. *BS 1377-90*; Methods of Test for Soils for Civil Engineering Purposes. Shear Strength Tests (total stress); Part 7. British Standard Institution: London, UK, 1990.
46. McLemore, V.T.; Sweeney, D.; Dunbar, N.; Heizler, L.; Writer, E.P. Determining quantitative mineralogy using a modified MODAN approach on the Questa rock pile materials, New Mexico. In Proceedings of the Society of Mining, Metallurgy and Exploration Annual Convention, Denver, CO, USA, 22–25 February 2009; pp. 9–20.
47. Zhai, Y.L.; Li, L.; Chapuis, R.P. Analytical, numerical and experimental studies on steady-state seepage through 3D rockfill trapezoidal dikes. *Mine Water Environ.* **2021**, *40*, 931–942. [CrossRef]
48. Azam, S.; Li, Q. Tailings dam failures: A Review of the Last One Hundred Years. *Geotech. News* **2010**, *28*, 50–54.
49. Owen, J.R.; Kemp, D.; Lèbre, É.; Svobodova, K.; Murillo, G.P. Catastrophic tailings dam failures and disaster risk disclosure. *Int. J. Disaster Risk Reduct.* **2020**, *42*, 101361. [CrossRef]
50. Aubertin, M.; Bussière, B.; Bernier, B. *Environnement et Gestion des Rejets Miniers*; Manuel sur Cédérom; Presses Internationales Polytechnique: Montreal, QC, Canada, 2002.
51. Azam, S.; Wilson, G.W.; Herasymuik, G.; Nichol, C.; Barbour, L.S. Hydrogeological behavior of an unsaturated waste rock pile: A case study at the Golden Sunlight Mine, Montana, USA. *Bull. Eng. Geol. Environ.* **2007**, *66*, 259–268. [CrossRef]
52. Boudrias, G. Évaluation Numérique et Expérimentale du Drainage et de la Consolidation de Résidus Miniers à Proximité d'une Inclusion de Roches Stériles. Master's Thesis, Polytechnique Montréal, Montreal, QC, Canada, 2018.
53. Saleh-Mbemba, F.; Aubertin, M.; Boudrias, G. Drainage and consolidation of mine tailings near waste rock inclusions. In *Sustainable and Safe Dams around the World*; Taylor & Francis Group: London, UK, 2019; pp. 3296–3305.
54. Li, L.; Ouellet, S.; Aubertin, M. A method to evaluate the size of backfilled stope barricades made of waste rock. In *GeoHalifax*; Canadian Geotechnical Society: Halifax, CA, Canada, 2009.
55. Li, L.; Aubertin, M. Limit equilibrium analysis for the design of backfilled stope barricades made of waste rock. *Can. Geotech. J.* **2011**, *48*, 1713–1728. [CrossRef]
56. Yang, P.Y.; Li, L.; Aubertin, M.; Brochu-Baekelmans, M.; Ouellet, S. Stability analyses of waste rock barricades designed to retain paste backfill. *Int. J. Geomech.* **2017**, *17*, 04016079. [CrossRef]
57. Zhai, Y.L.; Yang, P.Y.; Li, L. Analytical solutions for the design of shotcreted waste rock barricades to retain slurried paste backfill. *Constr. Build. Mater.* **2021**, *307*, 124626. [CrossRef]
58. Hall, E.B. A triaxial apparatus for testing large soil specimens. *ASTM Triaxial Test. Soils Bitum. Mix.* **1951**, *106*, 152–161.
59. Holtz, W.; Gibbs, H.J. Triaxial shear tests on pervious gravelly soils. *J. Soil Mech. Found. Div.* **1956**, *82*, 1–22. [CrossRef]
60. Leslie, D. Large scale triaxial tests on gravelly soils. In Proceedings of the Second Panamerican Conference on Soil Mechanics and Foundation Engineering, Sao Paulo, Brazil, 1 July 1963; Volume 1, pp. 181–202.
61. Marachi, N.; Seed, H.; Chan, C. Strength characteristics of rockfill materials. In Proceedings of the 7th International Conference on Soil Mechanics and Foundation Engineering, Mexico City, Mexico, 25–29 August 1969; pp. 217–224.

62. Marachi, N.D.; Chan, C.K.; Seed, H.B. Evaluation of properties of rockfill materials. *J. Soil Mech. Found. Div.* **1972**, *98*, 95–114. [CrossRef]
63. Varadarajan, A.; Sharma, K.G.; Venkatachalam, K.; Gupta, A.K. Testing and modeling two rockfill materials. *J. Geotech. Geoenviron. Eng.* **2003**, *129*, 206–218. [CrossRef]
64. Hamidi, A.; Azini, E.; Masoudi, B. Impact of gradation on the shear strength-dilation behavior of well graded sand-gravel mixtures. *Sci. Iran.* **2012**, *19*, 393–402. [CrossRef]
65. Chang, W.J.; Phantachang, T. Effects of gravel content on shear resistance of gravelly soils. *Eng. Geol.* **2016**, *207*, 78–90. [CrossRef]
66. Yang, G.; Jiang, Y.; Nimbalkar, S.; Sun, Y.; Li, N. Influence of particle size distribution on the critical state of rockfill. *Adv. Civ. Eng.* **2019**, *2019*, 1–7. [CrossRef]
67. Ovalle, C.; Linero, S.; Dano, C.; Bard, E.; Hicher, P.-Y.; Osses, R. Data compilation from large drained compression triaxial tests on coarse crushable rockfill materials. *J. Geotech. Geoenviron. Eng.* **2020**, *146*, 06020013. [CrossRef]
68. *ASTM D3080-72*; Direct Shear Test of Soils under Consolidated Drained Conditions. ASTM International: West Conshohocken, PA, USA, 1972.
69. Varadarajan, A.; Sharma, K.G.; Abbas, S.M.; Dhawan, A.K. The role of nature of particles on the behavior of rockfill material. *Soils Found.* **2006**, *46*, 569–584. [CrossRef]
70. Abbas, S.M. *Behavior of Rockfill Materials Based on Nature of Particles*; Lambert Academic Publishing: Saarbrücken, Germany, 2011.
71. Pankaj, S.; Mahure, N.; Gupta, S.; Sandeep, D.; Devender, S. Estimation of shear strength of prototype rockfill materials. *Int. J. Eng. Sci.* **2013**, *2*, 421–426.
72. Vasistha, Y.; Gupta, A.K.; Kanwar, V. Medium triaxial testing of some rockfill materials. *Electron. J. Geotech. Eng.* **2013**, *18*, 923–964.
73. Honkanadavar, N.P.; Kumar, N.; Ratnam, M. Modeling the behavior of alluvial and blasted quarried rockfill materials. *Geotech. Geol. Eng.* **2014**, *32*, 1001–1015. [CrossRef]
74. Xu, Y. Shear strength of granular materials based on fractal fragmentation of particles. *Powder Technol.* **2018**, *333*, 1–8. [CrossRef]
75. Dadkhah, R.; Ghafoori, M.; Ajalloeian, R.; Lashkaripour, G.R. The effect of scale direct shear test on the strength parameters of clayey sand in Isfahan city. *Can. J. Appl. Sci.* **2010**, *10*, 2027–2033. [CrossRef]
76. Deiminiat, A.; Li, L.; Zeng, F.; Pabst, T.; Chiasson, P.; Chapuis, R. Determination of the shear strength of rockfill from small scale laboratory shear tests: A critical review. *Adv. Civ. Eng.* **2020**, *2020*, 8890237. [CrossRef]
77. *ASTM C29/C29M-17a*; Standard Test Method for Bulk Density (unit weight) and Voids in Aggregate. ASTM International: West Conshohocken, PA, USA, 2007.
78. Hight, D.W.; Leroueil, S. Characterisation of soils for engineering purposes. *Characterisation Eng. Prop. Nat. Soils* **2003**, *1*, 255–360.
79. *ASTM D4767*; Standard Test Method for Consolidated Undrained Triaxial Compression Test for Cohesive Soils. ASTM International: West Conshohocken, PA, USA, 2011.