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New Approach to Solve Cubic Objective Function Programming Problem

Media A. Omer, Nejmaddin A. Sulaiman

Department of Mathematics, College of Education, Salahaddin University, Erbil, Iraq
Email: media.omer@su.edu.krd, Nejmaddin.Sulaiman@su.edu.krd

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Abstract

In this paper, a cubic objective programming problem (COPP) is defined. Introduced a new modification to solve a cubic objective programming problem. Suggested an algorithm for its solution. Also reported the algorithm of the usual simplex method. Application talks about how the developed algorithm can be used to unravel non-linear. The proposed technique, modification simplex technique, can be used with the constructed numerical examples an illustrative numerical problems are given to demonstrate the algorithms.

Keywords

New Approach, Cubic Objective Programming Problem, Simplex Method

1. Introduction

Nonlinear programming problems are mathematical programming problems with nonlinear/linear objective functions and linear/nonlinear constraints. There are several approaches for solving various sorts of non-linear programming problems that are affected by the kind of objective function and constraints in [1]. The number of methods with providing examples clearly discussed using standard division to sole multi-objective programming problem in [2]. [3] presented a specialization of the convex simplex method to cubic programming. [4] presented a method that's utilized to illuminate a set of nonlinear programming issues by simplex strategy. This technique also makes a difference to supply the arrangement of direct programming problems (Abdulrahim). Nonlinear optimization with financial application is been examined by [5]. Also, by utilizing altered simplex approach and Wolfes strategy QFPP illuminated by [6]. The cubic-quartic nonlinear Schrödinger and resonant nonlinear Schrödinger equation in parabolic law media are investigated to obtain the dark, singular, bright-singular combo and periodic soliton solutions by [7]. In 2020, A. M. Sultan *et al.* are studied solutions of higher order dispersive cubic-quantic nonlinear in [8] to broaden this work, they considered a unique case issue in which the target capacities are QF (Quadratic partial) however contain direct limitations. The issue will settle by another adjusted simplex strategy. Likewise, the issue of the extraordinary case will be tackled by simplex strategy after converting the target capacity to the pseudo partiality work. The two outcomes will be contrasted with test legitimacy. [9] discussed about linear and nonlinear operation research, named "Principles of Operations Research" in 1999.

Cubic objective programming problems (COPP) might be specified as a really critical point with respect to nonlinear programming. In expansion, direct programming is exceptionally vital for a few purposes counting (wellbeing care, generation and etc.) arranging. More specifically, in mentioned applications of nonlinear programming, two given portions or functions could be maximized and minimized.

In order to extend this work, we have defined a cubic objective programming problem with linear constraints (COPP) and suggested the algorithm to solve cubic programming problem; and proposed a new modification simplex method to find the solution of COPP.

2. Some Definition and Theorems

2.1. Linear Programming (LP)

The general linear programming model with n decision variables and m constraints can be stated in the following form.

Optimize (max or min) $Z = \sum_{i=1}^{n} C_i t_i$

Subject to

$$a_{11}t_1 + a_{12}t_2 + \cdots + a_{1n}t_n \begin{pmatrix} \ge \\ \le \\ = \end{pmatrix} b_1$$

$$a_{21}t_1 + a_{22}t_2 + \cdots + a_{2n}t_n \begin{pmatrix} \ge \\ \le \\ = \end{pmatrix} b_2$$

$$\vdots$$

$$a_{n1}t_1 + a_{n2}t_2 + \cdots + a_{nn}t_n \begin{pmatrix} \ge \\ \le \\ = \end{pmatrix} b_n$$

$$t \ge 0$$

where c_i represents the per unit profit (or cost) of decision variables $t_1, t_2, \cdots, t_n$ to the value of the objective function. And a_{ij} where $i = 1, 2, \cdots, n; j = 1, 2, \cdots, n$ represent the amount of resource consumed per unit of the decision variables. The b_i represents the total availability of the *i*th resource. *Z* represents the measure-of-performance which can be either profit, or cost or reverence etc.

2.2. Quadratic Programming

The optimization problems assume that form

$$\text{Max}\left(\text{Min}\right).\ Z = \alpha + C^{\mathrm{T}} t + t^{\mathrm{T}} G t$$

subject to:

$$At\begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix}b$$

$$t \geq 0$$

where $A = \left(a_{ij}\right)_{m\times n}$, matrix of coefficients.

For all $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$

$$b = \left(b_1, b_2, \cdots, b_n\right)^{\mathrm{T}},\ t = \left(t_1, t_2, \cdots, t_n\right)^{\mathrm{T}},\ Ct = \left(C_1, C_2, \cdots, C_n\right)^{\mathrm{T}}$$

And $G = \left(g_{ij}\right)_{n\times n}$ mentioned as a positive semi-definite organized four-sided matrix, also, the objective functions is quadratic and constraints are linear. So, shown problem could be expressed as a QP problem. For more details, see [3].

2.3. Nonlinear Programming Problem

The general non-linear programming problem can be stated in the following form: optimize

$$\text{Max}\left(\text{Min}\right).\ Z = \alpha + C^{\mathrm{T}} t + t^{\mathrm{T}} G t$$

subject to:

$$At\begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix}b$$

$$t \geq 0$$

where $A = \left(a_{ij}\right)_{m\times n}$, matrix of coefficients.

For all $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$

$$b = \left(b_1, b_2, \cdots, b_n\right)^{\mathrm{T}},\ t = \left(t_1, t_2, \cdots, t_n\right)^{\mathrm{T}},\ Ct = \left(C_1, C_2, \cdots, C_n\right)^{\mathrm{T}}$$

And $G = \left(g_{ij}\right)_{n\times n}$ mentioned as a positive semi-definite organized four-sided matrix, also, the objective functions is nonlinear and constraints are linear.

2.4. Theorem: Fundamental Theorem of LP

The ideal value of the target function in a LP issue exists, at that point that esteem (known as the ideal arrangement) or (optimal solution) should happen at least one of the limit points of the practical area [3].

3. Mathematical Form of COPP

The mathematical form of COPP cubic objective programming problem as follows:

$$\text{Max.}\ z = \sum_{i=1}^{n} C^{\mathrm{T}} t^{p}$$

subject to:

$$At(\leq,\geq,=)b$$
$$t\geq 0$$

where C is n-dimensional column vector, $p=1,2,3$, A is an $(m\times n)$ matrix and b is an m-dimensional column vector.

4. New Approach

In this paper the problem that has objective function from as follows

$$\text{Max. } z=a_1t_1^3+a_2t_1^2t_2+a_3t_1t_2^2+a_4t_2^3$$

subject to:

$$At\begin{pmatrix}\leq\\ \geq\\ =\end{pmatrix}b$$
$$t\geq 0$$

A is an $m\times n$ matrix, all vectors are assumed to be column vectors unless transposed (T), where t is an n-dimensional column vector of decision variables. a_1,a_2,a_3,a_4 are coefficients of objective functions, $t_1,t_2,\cdots,t_n$ are the value of objective functions.

$$b=(b_1,b_2,\cdots,b_n)^{\mathrm{T}},\quad t=(t_1,t_2,\cdots,t_n)^{\mathrm{T}}$$

5. Algorithms

5.1. Algorithm of Standard Division Technique to Solve COPP (Cubic Objective Programming Problem) of Form

$$\text{Max. } z=(a_1t_1+a_2t_2+\alpha)(b_1t_1+b_2t_2+\beta)(c_1t_1+c_2t_2+\gamma)$$

subject to:

$$At\begin{pmatrix}\leq\\ \geq\\ =\end{pmatrix}b$$
$$t\geq 0$$

A is an $m\times n$ matrix, all vectors are assumed to be column vectors unless transposed (T), where t is an n-dimensional column vector of decision variables, α, β, and γ are scalars.

Below algorithm shown to find the optimal average of maximum and minimum for the COPP as follows:

Step 1: Through clarifying and appearing slack and manufactured factors standard shape of the issue can be composed to limitations, and stamp starting simplex table.

Step 2: Compute the μ by through below equations

$$\mu=\min\left|\frac{t_B}{t_j}\right|$$

Step 3: Compute the Δ_j through below equations

$$\Delta_j = \left(Z_1\Delta_{j_2} + Z_2\Delta_{j_1}\right) + \left(Z_1\Delta_{j_3} + Z_3\Delta_{j_1}\right) + \left(Z_2\Delta_{j_3} + Z_3\Delta_{j_2}\right) + \mu\Delta_{j_1}\Delta_{j_2}\Delta_{j_3},$$

then mark or write computed value in the beginning simplex table.

Step 4: Get arrangement of to begin with objective issue through utilizing simplex way.

Step 5: Check the reply for attainability in step 4, in case of being doable go to step 6, and in case not, double simplex strategy will be utilizing in order to remove in feasibility.

Step 6: The arrangement for optimality will be check in the event that all $\Delta_j \geq 0$ at that point go to step 7, something else back to step 4.

Step 7: Dole out a title to ideal esteem of the greatest objective work Z_i say $\forall i = 1, 2, \cdots, r$ and allot a title to the ideal esteem of the most extreme objective work Z_i where $\forall i = r+1, r+2, \cdots, s$.

Step 8: Include overall objective functions through repeat procedure from the step 4: for $i = 2, \cdots, s$.

5.2. Algorithm and Solving Cubic Programming Problem by Modified Simplex Method

Cubic form as follows:

$$\text{Max. } z = a_1t_1^3 + a_2t_1^2t_2 + a_3t_1t_2^2 + a_4t_2^3$$

subject to:

$$At\begin{pmatrix}\leq \\ \geq \\ =\end{pmatrix}b$$

$$t \geq 0$$

A is an $m \times n$ matrix, all vectors are assumed to be column vectors unless transposed (T), where t is an n-dimensional column vector of decision variables.

5.3. Algorithm

1) $\max . z = \max . z_1 - \max z_2$. Then applying algorithm 4.1 to solve $\max . z_1$ and $\max z_2$.

2) Find $\max . z = \max . z_1 - \max z_2$.

6. Construct Numerical Example

Example 1

$$\text{Max. } Z = t_1^3 - 2t_1^2t_2 + 3t_1t_2^2 + t_2^3$$

subjected to:

$$t_1 + t_2 \leq 6$$

$$4t_1 - 2t_2 \leq 8$$

$$t_1, t_2 \geq 0$$

$$\max.z = t_1^2(t_1 - 2t_2) - t_2^2(-3t_1 - t_2)$$

subjected to:

$$t_1 + t_2 \le 6$$
$$4t_1 - 2t_2 \le 8$$
$$t_1, t_2 \ge 0$$

Then

$$\max.z_1 = t_1^2(t_1 - 2t_2)$$
$$\max.z_2 = t_2^2(-3t_1 - t_2)$$

subjected to:

$$t_1 + t_2 \le 6$$
$$4t_1 - 2t_2 \le 8$$
$$t_1, t_2 \ge 0$$

Solve each objective by the same constraints:

$$\max.z_1 = t_1^2(t_1 - 2t_2)$$

subjected to:

$$t_1 + t_2 \le 6$$
$$4t_1 - 2t_2 \le 8$$
$$t_1, t_2 \ge 0$$

where:

B_S is basic variables, CB_i is coefficient of basic variable in the objective function, $i = 1,2,3$. C_{j_1} is a coefficient of variables in the first factor of the objective function. C_{j_2} is a coefficient of variables in the second factor of the objective function. C_{j_3} is a coefficient of variables in the third factor of the objective function. t_B is a value of the basic variables, and z_1, z_2, z_3 value of the factors in the objective function, and $Z_1 = f_1 * f_2 * f_3$ value of the objective function.

$$f_1 = CB_1 * t_B = (0\ 0)(6\ 4) = 0,\ \ f_2 = CB_2 * t_B = (0\ 0)(6\ 4) = 0,$$
$$f_3 = CB_3 * t_B = (0\ 0)(6\ 4) = 0$$
$$f_1 = 0,\ \ f_2 = 0,\ \ f_3 = 0;\ \ Z_1 = f_1 * f_2 * f_3 = 0$$

Applying the procedure of simplex method, we get the optimal solution is $t_1 = 2$, $t_2 = 0$, $S_1 = 4$, $S_2 = 0$ and max. $Z = 8$.

where:

minratio = min{t_B/t_j, $t_j > 0$}, $\mu_j = \min\{t_B/t_j,\ t_j > 0\}$ for non-basic vectors, *i.e.* for $j = 1, 2$

$$\Delta_{j_1} = Z_1 * t_j - C_{j_1},\ \ \Delta_{j_2} = Z_2 * t_j - C_{j_2},\ \ \Delta_{j_3} = Z_3 * t_j - C_{j_3},$$

$C_{j_1} = (1\ 0\ 0\ 0)$ is a coefficients of variables in the first factor of the objective function, then $\Delta_{j_1} = (-1\ 0\ 0\ 0)$; $C_{j_2} = (1\ 0\ 0\ 0)$ is a coefficients of variables in the second factor of the objective function, then $\Delta_{j_2} = (-1\ 0\ 0\ 0)$ & $C_{j_3} = (1\ -2\ 0\ 0)$ is a coefficients of variables in the first factor of the objective function, then

$$\Delta_{j_3} = (1\ -2\ 0\ 0)\ \&\ \mu_1 = \{6/1, 2/1\} = 4;\ \mu_2 = \{6/1, \text{-}\} = 6$$

$$\Delta_j = (Z_1 * \Delta_{j_2} + Z_2 * \Delta_{j_1}) + (Z_1 * \Delta_{j_2} + Z_3 * \Delta_{j_1}) + (Z_2 * \Delta_{j_3} + Z_3 * \Delta_{j_2}) + \mu \Delta_{j_1} \Delta_{j_2} \Delta_{j_3}$$

then $\Delta_j = (-2\ 0\ 0\ 0)$

$$\max . z_2 = t_2^2 (-3t_1 - t_2)$$

subjected to:

$$t_1 + t_2 \le 6$$

$$4t_1 - 2t_2 \le 8$$

$$t_1, t_2 \ge 0$$

$$f_1 = 0,\ f_2 = 0,\ f_3 = 0;\ Z_2 = f_1 * f_2 * f_3 = 0$$

To apply simplex method in **Table 1**, since all $\Delta_j \ge 0$ then this solution is optimal $t_1 = 0$, $t_2 = 0$, $S_1 = 6$, $S_2 = 8$ and max. $Z_2 = 0$.

Leads to

To calculate Δ_j by the same manner in **Table 2**.

The symbols C_{j_1}, C_{j_2}, C_{j_3}, Δ_{j_1}, Δ_{j_2}, Δ_{j_3}, Δ_j and μ_j have the same meaning as before in **Table 2**.

$$\mu_1 = \{6/1, 8/4\} = 2,\ \mu_2 = \{6/1, \text{-}\} = 6$$

$$C_{j_1} = (0\ 1\ 0\ 0),\ C_{j_2} = (0\ 1\ 0\ 0),\ C_{j_3} = (-3\ -1\ 0\ 0)$$

$$\Delta_{j_1} = (0\ -1\ 0\ 0),\ \Delta_{j_2} = (0\ -1\ 0\ 0),\ \Delta_{j_3} = (3\ 1\ 0\ 0),$$

Table 1. First table of modification simplex method for solving cubic objective function.

B_s	CB_1	CB_2	CB_3	t_B	t_1	t_2	S_1	S_2	Min ratio
S_1	0	0	0	6	1	1	1	0	6/1 = 6
S_2	0	0	0	8	4	−2	0	1	8/4 = 2
				Δ_j	0	4	0	0	

Table 2. First table of modification simplex method for solving cubic objective function.

B_s	CB_1	CB_2	CB_3	t_B	t_1	t_2	S_1	S_2	Min ratio
S_1	0	0	0	6	1	1	1	0	6/1 = 6
S_2	0	0	0	4	1	−2	0	1	4/1 = 4
				Δ_j	−2	0	0	0	

$$\Delta_j = \begin{pmatrix} 0 & 4 & 0 & 0 \end{pmatrix}.$$

Then we get the value of objective function Z as:

$$\text{Max } Z = \text{Max. } Z_1 - \text{Max. } Z_2 = 8 - 0 = 8$$

The solution is $\text{Max } Z = 8 - 0 = 8$

Example 2:

$$\text{Max. } Z = 8t_1^3 - 2t_1t_2^2 + 3t_2^3$$

subjected to:

$$-4t_1 + 3t_2 \le 12$$

$$5t_1 + 3t_2 \le 15$$

$$t_1, t_2 \ge 0$$

$$\max.\, z = 8t_1^3 - t_2^2\left(2t_1 - 3t_2\right)$$

subjected to:

$$-4t_1 + 3t_2 \le 12$$

$$5t_1 + 3t_2 \le 15$$

$$t_1, t_2 \ge 0$$

Then

$$\max.\, z_1 = 8t_1^3$$

$$\max.\, z_2 = t_2^2\left(2t_1 - 3t_2\right)$$

subjected to:

$$-4t_1 + 3t_2 \le 12$$

$$5t_1 + 3t_2 \le 15$$

$$t_1, t_2 \ge 0$$

Solve each objective by the same constraints:

$$\max.\, z_1 = 8t_1^3 = 2t_1 * 2t_1 * 2t_1$$

subjected to:

$$-4t_1 + 3t_2 \le 12$$

$$5t_1 + 3t_2 \le 15$$

$$t_1, t_2 \ge 0$$

where:

B_S is basic variables, CB_i is coefficient of basic variable in the objective function, $i = 1,2,3$. C_{j_1} is a coefficient of variables in the first factor of the objective function. C_{j_2} is a coefficient of variables in the second factor of the objective function. C_{j_3} is a coefficient of variables in the third factor of the objective function. t_B is a value of the basic variables, and z_1, z_2, z_3 value of the factors in the objective function, and $Z_1 = f_1 * f_2 * f_3$ value of the objective function.

$$f_1 = 0,\ \ f_2 = 0,\ \ f_3 = 0;\ \ Z_2 = f_1 * f_2 * f_3 = 0$$

Applying the procedure of simplex method, we get the optimal solution is t_1 = 3, t_2 = 0, s_1 = 24, s_2 = 0 and max. Z = 216.

where:

To calculate Δ_j by the same manner in **Table 2**.

The symbols C_{j_1}, C_{j_2}, C_{j_3}, Δ_{j_1}, Δ_{j_2}, Δ_{j_3}, Δ_j and μ_j has the same meaning as before in **Table 2**

$$\mu_1 = \{\text{-}, 15/5\} = 3,\ \ \mu_2 = \{12/3, 15/3\} = 4$$

$$C_{j_1} = (2\ 0\ 0\ 0),\ C_{j_2} = (2\ 0\ 0\ 0),\ C_{j_3} = (2\ 0\ 0\ 0)$$

$$\Delta_{j_1} = (-2\ 0\ 0\ 0),\ \Delta_{j_2} = (-2\ 0\ 0\ 0),\ \Delta_{j_3} = (-2\ 0\ 0\ 0).$$

$$\Delta_j = (-24\ 0\ 0\ 0)$$

The optimal solution is t_1 = 3, t_2 = 0, S_1 = 24, s_2 = 0 and $\text{Max.}\ Z_1 = 6*6*6 = 216$

Now to solve max. Z_2 as:

$$\max.\ z_1 = t_2^2 (2t_1 - 3t_2)$$

subjected to:

$$4t_1 - 3t_2 \le 12$$

$$5t_1 + 3t_2 \le 15$$

$$t_1, t_2 \ge 0$$

where:

All the symbols have the same meaning as before in **Table 2**, **Table 3**.

$$f_1 = 0,\ \ f_2 = 0,\ \ f_3 = 0;\ \ Z_2 = f_1 * f_2 * f_3 = 0$$

applying the procedure of simplex method in **Table 4**, since all $\Delta_j \ge 0$ then this solution is optimal t_1 = 0, t_2 = 0, S_1 = 12, S_2 = 15 and Max. Z_2 = 0.

where

To calculate Δ_j by the same manner in **Table 2**

$$\mu_1 = \{6/1, 8/4\} = 2,\ \ \mu_2 = \{6/1, \text{-}\} = 6$$

$$C_{j_1} = (0\ 1\ 0\ 0),\ C_{j_2} = (0\ 1\ 0\ 0),\ C_{j_3} = (-2\ 3\ 0\ 0)$$

$$\Delta_{j_1} = (0\ -1\ 0\ 0),\ \Delta_{j_2} = (0\ -1\ 0\ 0),\ \Delta_{j_3} = (2\ -3\ 0\ 0).$$

$$\Delta_j = (0\ 12\ 0\ 0)$$

Then we get the value of objective function Max. Z as:

$$\text{Max}\ Z = \text{Max.}\ Z_1 - \text{Max.}\ Z_2 = 216 - 0 = 216$$

The solution is $\text{Max}\ Z = 216 - 0 = 216$

In **Table 5**, it is clear that the results optioned in examples, which solved by modification simplex method.

Table 3. First table of modification simplex method for solving cubic objective function.

B_s	CB_1	CB_2	CB_3	t_B	t_1	t_2	S_1	S_2	Min ratio
S_1	0	0	0	12	−4	3	1	0	-
S_2	0	0	0	15	5	3	0	1	15/3 = 5
				Δ_j	−24	0	0	0	

Table 4. First table of modification simplex method for solving cubic objective function.

B_s	CB_1	CB_2	CB_3	t_B	t_1	t_2	S_1	S_2	Min ratio
S_1	0	0	0	12	−4	3	1	0	-
S_2	0	0	0	15	5	3	0	1	-
				Δ_j	0	12	0	0	

Table 5. Results of the numerical approaches.

Example 1	t_1	t_2	s_1	s_2	z_1	z_2	z
	2	0	4	0	8	0	8
Example 2	t_1	t_2	s_1	s_2	z_1	z_2	z
	3	0	24	0	216	0	216

7. Conclusions

In this paper, we try to draw certain conclusions based on our experience of working with the algorithm developed in this paper. The algorithm developed in this paper found computationally efficient to solve the related type of cubic programming problems.

In fact, the related the theoretical development of algorithm is useful only when their computer programs are available for quick and accurate solution of practical problems of large dimensions.

In this work example 1 showed that the solution of cubic programming problem, $t_1 = 2$, $t_2 = 0$, and $\max. z_1 = 8$, $\max. z_2 = 0$, and $\max. z = 8$, similarly in example 2 $t_1 = 3$, $t_2 = 0$, and $\max. z_1 = 216$, $\max. z_2 = 0$, and $\max. z = 216$.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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