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Using Singular Value to Set Output Disturbance Limits to Feedback ILC Control

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Abstract

Iterative Learning Control is an effective way of controlling the errors which act directly on the repetitive system. The stability of the system is the main objective in designing. The Small Gain Theorem is used in the design process of State Feedback ILC. The feedback controller along with the Iterative Learning Control adds an advantage in producing a system with minimal error. The past error and current error feedback Iterative control system are studied with reference to the region of disturbance at the output. This paper mainly focuses on comparing the region of disturbance at the output end. The past error feed forward and current error feedback systems are developed on the singular values. Hence, we use the singular values to set an output disturbance limit for the past error and current error feedback ILC system. Thus, we obtain a result of past error feed forward performing better than the current error feedback system. This implies greater region of disturbance suppression to past error feed forward than the other.

Keywords

Iterative Learning Control, Disturbance Output, Singular Values

1. Introduction

Learning control is an effective tool in the field of control. The combination of learning control along with artificial intelligence provides much new advancement in the field of robotics, manufacturing and transportation [1] [2]. Among a variety of learning control techniques, Iterative Learning control ILC arises and executes the same control task repeatedly with finite time duration [3]. Feedback control system states that, “the system whose output is controlled using its measurement as a feedback signal” [4]. The feedback signal is compared to the reference signal in order to generate an error signal which is filtered by a controller and in turn produces the system's controlled input.

Feedback ILC is a well known controlling method to enhance the performance of system in repetitive mode. The idea of an ILC is to build up a series of controlling input (u_k) such that the error (e_k) tends to decay on repeated iterations or to an acceptable error tolerance [5]. A trial in ILC represents a complete task for predefined time duration. A reference ($r(t)$) is assumed to have a time duration governed by $0 \le t \le T < \infty$, in which T represents the length of the trial. Once a process is completed, the data fetched is available to reckon the control input for the following iterative process. Robot manipulators perform repetitive operation of pick and place at finite duration. Gantry robot application is used to collect an object from a fixed location point and transfer it to another location within a predefined period [6]. Then the robot returns to its normal position of start to perform the specified task repeatedly. The aim is to perform a predefined task repeatedly as many times as possible, without the need for resetting. Similar operations are performed in other applications like Microelectronics manufacturing, chemical batch processes and petrochemical processes. The integer $k \ge 0$ shows the trial number and $y_k(t)$ the end result on trial k. Here we focus on how to limit the single-input-single-output systems with universality to multi-input multi-output systems (MIMO). Furthermore, the error on trial k is $e_k(t) = r(t) - y_k(t)$.

With the presence of previous trial information, the current trial input can be formed as non-causal temporal information.

The early development of ILC was reported by [5]; a derivative type of ILC is introduced as $u_{k+1} = u_k + \Upsilon \dot{e}_k$, where Υ is the learning gain. Since then, an extensive effort was taken to introduce several developments on ILC, see [7] for example.

There are two types of ILC development, one based on the presence of system dynamics matrix. The other is based on the development of control input law where the dynamic matrix is excluded, such as the phase-lead ILC [8]. The latter case suffers from lack of control performance, thus the first comes as good solution.

This paper considers completing the disturbance scenario of current error feedback state ILC depending on [9] and [10]. The introduction work considers repeated disturbance acting on system input. The work in [11] presents modified work that includes past and current error feedback ILC.

Uncertainty in control is a common issue to investigate, as well as disturbances. Several reported works investigate the above issues as [12] [13]. [13] for example, discusses load disturbance for state feedback ILC in past error feed forward. [14] gave an extended work to the ILC design for the current error feedback and past error feed forward by adding the external instability conditions on the load.

This paper investigates the output disturbance condition for past and current error feedback. Several conditions are erected to ensure system stability and performance enhancement. This shows a developed system for load disturbance as in [14].

Further we revise [11], and then new conditions are obtained for past error and current error feedback. Finally, a conclusion is given, and a possible future work is clarified.

2. Background

Initially we edit the ILC design initiated in [11], by taking a linear MIMO system S of m outputs, p inputs and n states. The state form $S(z)=C(zIn-F)-1\Xi+D$ elaborates the complete transfer function in the state space form at discrete time-invariant. The matrices f, Ξ, C and D are the proportions which helps the previously mentioned equation vital.

The design input of size $p\times o$, is $u(z)$ and the output of size $m\times o$, is $y(z)$. Thus, the output $y(z)$ can be represented as $y(z)=S(z)u(z)$. The known fact of ILC is that, the design processes a single trial in a defined time and after its goes back to its initial state for the next trial to be started. A single trial with a pre-determined time can be used to show a system dynamics over a single trial. This is illustrated as

$$\begin{aligned} x_k(i+1) &= Fx_k(i)+\Xi u_k(i),\ \ x_k(0)=x_0 \\ y_k(i) &= Cx_k(i)+Du_k(i), \end{aligned} \tag{1}$$

In the above Equation (1), $0\le i\le N-1$ where N is the number of trials. Because of the resetting condition used, it is well appreciated to take the first value $X_o=0$. The Equation (1) is formatted in two different dimensions, one of which is reflected earlier at the initialization of ILC for continuous time domain and discrete field. The other creates an essential base to the ILC interest, due to its character of sorting data. Many ILC models are completely depending on changing the discrete illustration as an index trial notation which is a one notational form, see [7]. Hence, the modified statement begins with including the input and output super vectors; u and y respectively on the trial index

$$u_k=\left[u_k(0),u_k(1),\cdots,u_k(N-1)\right]^{\mathrm{T}}$$

$$y_k=\left[y_k(0),y_k(1),\cdots,y_k(N-1)\right]^{\mathrm{T}}$$

System stability is a keynote criterion for ILC design systems; so a response connection is established to balance the iterative process. Hence the overall dynamics can be expressed as

$$y_k=u_k \tag{2}$$

where S denotes a lower triangular Toeplitz matrix. The down parameters in the matrix are Markov parameters, which can be shown as

$$S=\begin{bmatrix} C\Xi & 0 & 0 & \cdots & 0 \\ CF\Xi & C\Xi & 0 & \cdots & 0 \\ CF^2\Xi & CF\Xi & C\Xi & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CF^{N-1}\Xi & Cf^{N-2}\Xi & CF^{N-3}\Xi & \cdots & C\Xi \end{bmatrix}$$

To keep the vector form in discrete space, the reference $r(t)$ is defined as

$$r = \left[r(0), r(1), \cdots, r(N-1)\right]^{\mathrm{T}}$$

In a process of measuring an inaccuracy, the ILC system uses a predefined inaccurate consonant as a forcing function which is indulged to the old iterative input to produce the consecutive iteration input signal. This design follows the reference trajectory precisely along with the trial index as it moves towards infinity.

[15] illustrated a periodic signal of length N which is described in the discrete-time formation as

$$\begin{gathered} x_w(t_k+1) = Fx_w(t_k),\ \ x_w(t0) = x_w 0 \\ w(t_k) = C_w x(t_k), \end{gathered} \tag{3}$$

The $N \times N$ matrix F_w is shown as

$$F_w = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

The row vector C_w of the size $1 \times N$ is shown as

$$C_w = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

The control issue in ILC state feedback model can be elaborated further. We need to identify the robust controller $K(z)$, where the Z denotes the discrete-time delay operator. For a robust periodic control problem, a transfer-function matrix $S(z)$ with the size of $m \times p$, which has an input vector consisting of a plant and a disturbance input $u = u_s + u_w$. Whereas the output signal (2) is illustrated and also a reference signal $r(t_k) = r(t_{k+N})$, $t_k = 0, \Delta T, 2\Delta T, \cdots$ with a sampling time of N.

The main focus is to create a controller $K(z)$ in a way that, the full closed-loop system is completely stable without any conditions. Henceforth the tracking error $e_k = r - y_k$ is zero along with the trial domain thus; the two rules are firmly stable.

To create an ILC controller in several design schemes, [11] extended the design reported by [9] [10]. The first one is with the state feedback.

$$\tilde{u}(i) = -K_l \begin{bmatrix} x_{l,k}(i) \\ x_k(i) \end{bmatrix}$$

And the second was through output injection. Both design schemes have variable stability conditions and it depends on the design scheme whether it uses current error feedback or past error feed forward. Thus, this balancing condition is attained

$$\left\| H(z) \right\| < 1 \tag{4}$$

In this research we consider only the feedback case. For which $H(z)$ is the overall transfer function around the delay model, $S(z)$ is the plant model [9] and $G(z)$ is the comprehensive transfer function of the system. Previously for the feed forward model, the stability Condition is

$$H(z)=\left(G(z)+S(z)\right)G(z)^{-1} \tag{5}$$

The stability condition of feedback model is

$$H(z)=G(z)\left(G(z)+S(z)\right)^{-1} \tag{6}$$

In both the cases of feed forward and feedback type of models, $G(z)$ is

$$G(z)=\begin{bmatrix}DC_l & C\end{bmatrix}\left(zI_{N_{p+n}}-\begin{bmatrix}F_l & 0\\ \Xi C_l & F\end{bmatrix}+\begin{bmatrix}\Xi_l\\ D\Xi_l\end{bmatrix}K_l\right)^{-1}\begin{bmatrix}\Xi_l\\ D\Xi_l\end{bmatrix}+DD_l.$$

Earlier the design implemented in [11] did not consider the scenario at which, the disturbance may act on the system load. And the design implemented in [13] included the scenario with past error feed forward only. Here we compare the current error feedback with [13], which is the past error feed forward.

3. Output Disturbance Limitation in Singular Values for State-Feedback ILC

Initially, [13] explained the system illustrated in (1) which is a single-input single-output condition in terms of load and measurement disturbances $d_k(t)$ and $n_k(t)$ as

$$\begin{aligned}&\Psi_k(t+\delta)=S(q)u_k(t)+d_k(t)\\ &y_k(t)=\Psi_k(t)+n_k(t),\ t=0,1,\cdots,n-1,\end{aligned} \tag{7}$$

The term k and q define the iteration index and forward shift operator respectively. The output illustration includes the time delay operator. Since there is no loss of generality, we take over that there is no delay with the process matrix $S(q)$. In the start of each iterative process, the model is programmed to initiate from a stable position. The count of sample in a trial process is $N+\delta$.

Consider, if a control action takes place at time $t=0$, the system will respond when $t=\delta$. Hence, it is insignificant to control the output $\Psi_k(t)$ at times $\delta\le t\le\delta+N-1$, with the input $u_k(t)$ at times $0\le t\le N-1$ as well as the measured output $y_k(t)$. The reference signal $r(t)$ is illustrated over a range of $\delta\le t\le\delta+N-1$, and the control issue might allow $\Psi_k(t)$ and $y_k(t)$ to follow $r(t)$ closely, where $r(t)$ remain unchanged during the complete trial process.

The model in (7) can be explained using the control input signal $u_k(t)$ which is mentioned earlier. The result $\Psi_k(t)$ for trial k is shown as

$$\Psi_k=\left[\Psi_k(\delta),\Psi_k(\delta+1),\cdots,\Psi_k(\delta+N-1)\right]^{\mathrm{T}} \tag{8}$$

The measured output $y_k(t)$, is defined similarly. The load disturbance vector d_k is parallel to $u_k(t)$. Whereas the measurement disturbance n_k, the measured output vector y_k, and the reference vector r are illustrated similar to (8).

Much needed presumptions made about d_k and n_k is as follows: 1) their mean is zero, weakly stationary random variables with bounded variance; 2) they are in phase with one another; and 3) they are collinear in-between trials.

Before analyzing output disturbance limitation conditions, we take into account the disturbance limitation to guarantee the system performance. A stable condition (5) for the state feedback design with past error feed forward and the output (7) creates a path using singular values as, a high confining region would result in the following condition as it was obtained in [14]:

$$\bar{\sigma}(d_k) < \bar{\sigma}\left(\sum_{i=0}^{k}(\Psi_i) - \sum_{j=0}^{k-1}(d_j) - Gu_0\right) - \underline{\sigma}(Gu_{k-1}) \quad (9)$$

This condition will be the guidance to form the new output disturbance condition that assures system stability in front of output disturbances acting on the system output. Consider the equation which led to (9),

$$\sigma(\Psi_k - d_k + Gu_{k-1} + \Psi_k - d_k) < \sigma(\Psi_k - d_k + Gu_{k-1})$$

Adding the measurement part to the equation will lead to,

$$\begin{aligned}&\sigma\left(\Psi_k - d_k + n_k(t) - n_k(t) + Gu_{k-1} + \Psi_k - d_k + n_k(t) - n_k(t)\right)\\&< \sigma\left(\Psi_k - d_k + n_k(t) - n_k(t) + Gu_{k-1}\right)\end{aligned}$$

This can be written as,

$$\sigma(y_k - d_k - n_k(t) + Gu_{k-1}) < \sigma(Gu_{k-1})$$

This leads to form the further illustration as,

$$\bar{\sigma}(n_k) < \bar{\sigma}\left(\sum_{i=0}^{k}(y_i) - \sum_{j=0}^{k-1}(d_j) - Gu_0\right) - \underline{\sigma}(Gu_{k-1}) \quad (10)$$

Above illustration makes clear that the maximum singular value of the output interference implying on the current iteration has to be minimum than the maximum singular value of the difference of the summation of all previous iteration results eigen value further subtracting the sum of past iterative load changes, the first input feedback and the minimum singular value to the concluding iterative control response. Hence, the sweep where the output interference implicating on any trial k is highly confining and has a minute deviation with regards to its maximum singular value.

For current error feedback, the output interference restriction problem is obtained in a similar format. Initially to begin with the stability condition as shown in (6). The load disturbance may occur at any occasion in trial k and it is non-repetitive as well as the output disturbance. So it should be in shape that includes its weight of direction such that its result is examined and contained. Hence considering singular value analysis, the maximum singular value illustrating the interference should be confined at a stable region. The investigation including the singular value will lead to a conclusive illustration as

$$\bar{\sigma}\left(\sum_{i=0}^{k}(y_i) - \sum_{j=0}^{k-1}(d_j) - Gu_0\right) - \underline{\sigma}(Gu_{k-1}) < \bar{\sigma}(n_k) \quad (11)$$

And this can be rewritten as

$$\bar{\sigma}\left(\sum_{i=0}^{k}\left(y_i\right)-\sum_{j=0}^{k-1}\left(d_j\right)-Gu_0\right)-\bar{\sigma}\left(\sum_{h=0}^{k-1}\left(y_h\right)-\sum_{v=0}^{k-1}\left(d_v\right)-Gu_0\right) < \bar{\sigma}\left(n_k\right)<1 \quad (12)$$

The conclusive illustration (12) clearly defines that; the highest singular value of the output interference should always be higher than the sum of a complete iterative output singular value. And it also should not involve the least singular value of the sum of previous conclusive signals, previous interferences, first output, the highest singular value of previous interference and First output. And also the expression should not be greater than 1. As it was pointed out in [14], it is very hard to attain the desired result with a feeble resource like past error feedforward which involves a highly attainable region of interference discretion.

The output (12) states firmly that the positivity of the previous error feed forward is because of its compact structure, feasible stability conditions and output disturbance limitation conditions.

4. Conclusion

Past error and current error feedback ILC schemes have been revisited. Output disturbance condition has been introduced in both cases. The results obtained verify the superiority of the previous error feed forward over current error feedback. This is achieved because of the obtained region of disturbance suppression. As it is shown, the previous error feed forward case is having greater suppression in the region of disturbance when correlated with current terror feedback. In future, a simulation model will be designed along with, the reader might join all developed dis-condition, uncertainty condition, and control law development in one reported work to present a complete design.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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