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# Theoretical Study on Spherical Composite Accelerating Cavity

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## Abstract

A composite accelerating cavity utilizing a resonant, periodic structure with a dielectric sphere located at a spherical conducting cavity center is presented. The resonator design is of the whispering gallery type to take advantage of the excellent electromagnetic field confinement offered by this geometry. The maximum electromagnetic fields of this structure exceed by several orders of magnitude the values reached in resonant cavities of typical linear accelerators. And the skin current losses are reduced without engaging superconductivity and cryogenic systems for this new construction. Especially because all field components at the metallic wall are either zero or very small in this proposed spherical cavity, one can expect the cavity to be less prone to electrical breakdowns than the traditional cavity. In this paper, the new type of accelerating structure was analyzed and calculated. The results are in very well agreement with the corresponding values simulated by CST. And for the existing ultra-low loss dielectrics, $Q$ can be three orders of magnitude better than obtained in existing cylindrical cavities.

## Keywords

High Quality, TE Mode, Composite Spherical Cavity

## 1. Introduction

With the development of high energy physics, medicine, materials science and molecular biology, the urgent needs of effectively promoting new accelerate prin-

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ciple and the study of the method of new acceleration ask a new generation of compact accelerator with high gradient and low losses.

At present, we use microwave to accelerate particles usually, but as we know, the high energy stored in the microwave cavity is limited by the metal surface breakdown and power losses at conducting cavities and waveguides. In recent years, in the areas of advanced accelerator research and development, there have been many methods which have been proved to accelerate the electron to very high energy. For example, laser wake-field accelerator (LWFA [1]), plasma wake-field accelerator (PWFA [2]), plasma beat-wave accelerator (PBWA) [3], dielectric wake-field accelerator (DWA) [4], inverse free electronic laser (IFEL) [5], inverse Cherenkov accelerator (ICA) [6], the vacuum laser accelerators(VLA) [7], and so on. But all the above need a very high energy laser beam(magnitudes of TW) and are limited to very short distances because of keeping synchronous.

In this paper, a composite resonator is presented for linacs, whose accelerating units are comprised of periodically shaped cavities with a smaller dielectric sphere located at a spherical conducting cavity center [8]. See **Figure 1**. We know that when a plane wave is scattered on a limited size dielectric object, the structural resonances can be excited in the object, e.g., whisper gallery modes (WGM). Because of the resonance, the amplitudes of the electromagnetic fields in the dielectric sphere and its vicinity are very high, the maximum value exceeds by several orders of that in typical resonant linear accelerators. A particular example of the WGM resonance shows that the accelerating fields are almost 100 times stronger than those in the incident wave [9]. So take usage of this structure to accelerate charged particles is very effective.

For the conventional accelerators, the strength of fields for the conventional accelerators is limited by electric breakdown and power losses at conducting cavities and waveguides. Compared to the existing particle accelerators, the skin current losses are reduced without engaging superconductivity and cryogenic systems for this new construction. Especially because all field components at the metallic wall are either zero or very small in this proposed spherical cavity through our simulation, one can expect the cavity to be less prone to electrical breakdowns than the traditional cavity. In the following sections a new type of accelerating structure was analyzed and calculated.

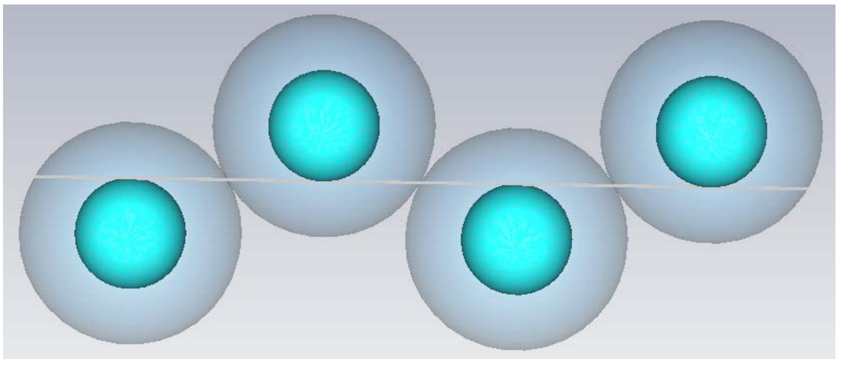

**Figure 1.** Proposed multi-cell accelerator units.

## 2. Simulation and Calculation

For the composite resonator structure, the accelerating fields had been discussed in standard textbooks on high energy particle accelerators, e.g. Jackson Classic Electrodynamics [10]. Because the structure is symmetric, the TE field inside a spherical cavity with a dielectric sphere in the spherical coordinate system has the following form

$$\begin{cases} \boldsymbol{E}^{lm}(\boldsymbol{r},t) = \boldsymbol{E}^{lm}(\boldsymbol{r},\omega)\mathrm{e}^{-i\omega t} & \\ \boldsymbol{E}^{lm}(\boldsymbol{r},\omega) = E_t(kr)\boldsymbol{X}^{lm}(\theta,\phi), & (1) \\ \boldsymbol{B}_t^{lm}(\boldsymbol{r},t) = \boldsymbol{B}_t^{lm}(\boldsymbol{r},\omega)\mathrm{e}^{-i\omega t} & \\ \boldsymbol{B}_t^{lm}(\boldsymbol{r},\omega) = B_t(kr)\boldsymbol{n}\times\boldsymbol{X}^{lm}(\theta,\phi), & (2) \\ \boldsymbol{B}_r^{lm}(\boldsymbol{r},t) = \boldsymbol{B}_r^{lm}(\boldsymbol{r},\omega)\mathrm{e}^{-i\omega t} & \\ \boldsymbol{B}_r^{lm}(\boldsymbol{r},\omega) = B_r(kr)\boldsymbol{n}Y^{lm}(\theta,\phi), & (3) \end{cases}$$

where $k = \omega/c$, and $\omega$ is the angular frequency, $(r,\theta,\phi)$ are spherical coordinates, the complex dielectric permittivity is $\epsilon$, $Y^{lm}(\theta,\phi)$ is spherical harmonics and $\boldsymbol{X}^{lm}(\theta,\phi)$ is vector spherical harmonics as defined in Jackson's Classical Electrodynamics [10], $\boldsymbol{n} = \boldsymbol{r}/r$, and $l$ is a positive integer related to the integer $m$ by $-l \le m \le l$.

Considering boundary conditions that the tangent component of the electric field must be zero at the ideal metallic wall and must be regular at the cavity center, from Jackson's Classical Electrodynamics we can assume that the form of the tangent component is as follows [8]:

$$E_t(kr) = N_l \times \begin{cases} A_l j_l\left(\sqrt{\epsilon}kr\right) & 0 \le r < a \\ \dfrac{j_l(kr)}{j_l(kb)} - \dfrac{y_l(kr)}{y_l(kb)} & a \le r \le b \end{cases} \tag{4}$$

wherein $j_l(\rho) \equiv \sqrt{\frac{\pi}{2\rho}} J_{l+\frac{1}{2}}(\rho)$ and $y_l(\rho) \equiv \sqrt{\frac{\pi}{2\rho}} Y_{l+\frac{1}{2}}(\rho)$ are spherical Bessel and Neumann functions, and $a$ and $b$ are radius of the dielectric sphere and the outer cavity.

From Maxwell's equation, we know

$$\nabla \times \boldsymbol{E} = ik\boldsymbol{B} \tag{5}$$

From Equations (1) (4) and (5) we get

$$B_t(kr) = \frac{-iN_l}{kr} \times \begin{cases} A_l j_l^D\left(\sqrt{\epsilon}kr\right) & 0 \le r < a \\ \dfrac{j_l^D(kr)}{j_l(kb)} - \dfrac{y_l^D(kr)}{y_l(kb)} & a \le r \le b \end{cases} \tag{6}$$

$$B_r(kr) = \frac{\sqrt{l(l+1)}}{kr} E_t(kr) \tag{7}$$

where $j_l^D(\rho) = \frac{\mathrm{d}}{\mathrm{d}\rho}\left(\rho j_l(\rho)\right)$ and $y_l^D(\rho) = \frac{\mathrm{d}}{\mathrm{d}\rho}\left(\rho y_l(\rho)\right)$ are derivatives of the

Riccati-Bessel and Riccati-Neumann functions.

Using the fact that the tangent electric field and magnetic induction must be continuity at the boundary of the dielectric sphere, we can infer the following equation

$$j_l\left(\sqrt{\epsilon}ka\right)\left[\frac{j_l^D\left(ka\right)}{j_l\left(kb\right)}-\frac{y_l^D\left(ka\right)}{y_l\left(kb\right)}\right]-j_l^D\left(\sqrt{\epsilon}ka\right)\left[\frac{j_l\left(ka\right)}{j_l\left(kb\right)}-\frac{y_l\left(ka\right)}{y_l\left(kb\right)}\right]=0 \quad (8)$$

From this equation, we can inform the eigen-frequencies of the cavity modes $\omega=\omega^l$ from $k=k^l$. And with the choice of $k^l$, $A_l$ can be obtained. On the basis of the above equations, assuming the electrons transit length through the cavity is the same as that of SLAC pill box cavity $d$ = 4 cm, we take four groups of $a$ and $b$ which satisfy the electrons transit length in the composite resonator $d$ = 4 cm to calculate the resonant frequencies $f(l)$ with MATLAB, the result is shown in Figure 2.

Figure 3 shows that all field components at the metallic wall are either zero or very small, so one can expect the cavity to be less prone to electrical breakdowns than the traditional cavity.

Let $a$ = 0.708 cm, $b$ = 2.124 cm, we simulated the electric field gradient distribution curve along the direction of electron motion, see Figure 4. we observe that the maximum value of the electric field is distribute very closely vicinity of the dielectric sphere.

Figure 5 is the tangential electric field along the radial distribution, which is simulated with CST, it shows that the result match very well with that of the calculation with Matlab.

Figure 6 shows the simulated result of the electric field distribution in the composite resonator, we can see the electric field along the trajectory is very large.

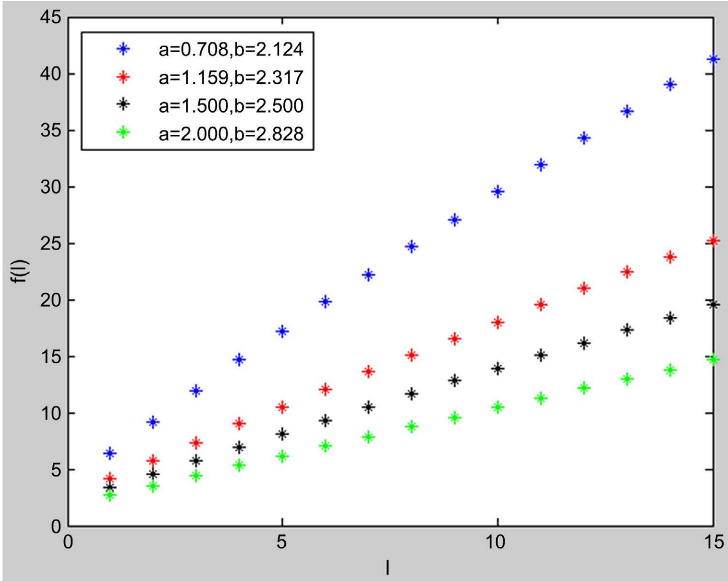


Figure 2. The resonant frequencies $f(l)$ for four spherical cavities with dielectric spheres.

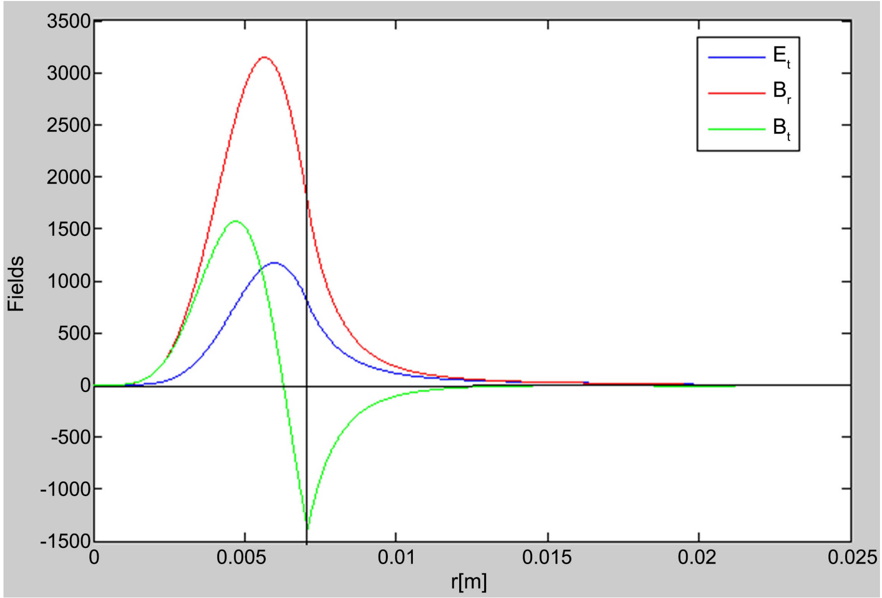


**Figure 3.** Field radial functions for a spherical cavity with dielectric sphere and perfect metallic wall ($a = 0.00708$ m, $b = 0.02124$ m, $\epsilon = 10$, $l = 7$, $k_0 = 413.407$).

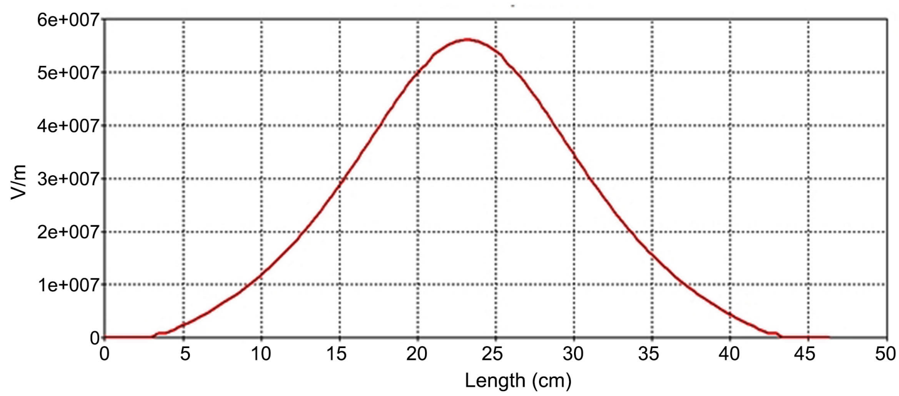


**Figure 4.** The electric field distribution curve along the direction of electron motion.

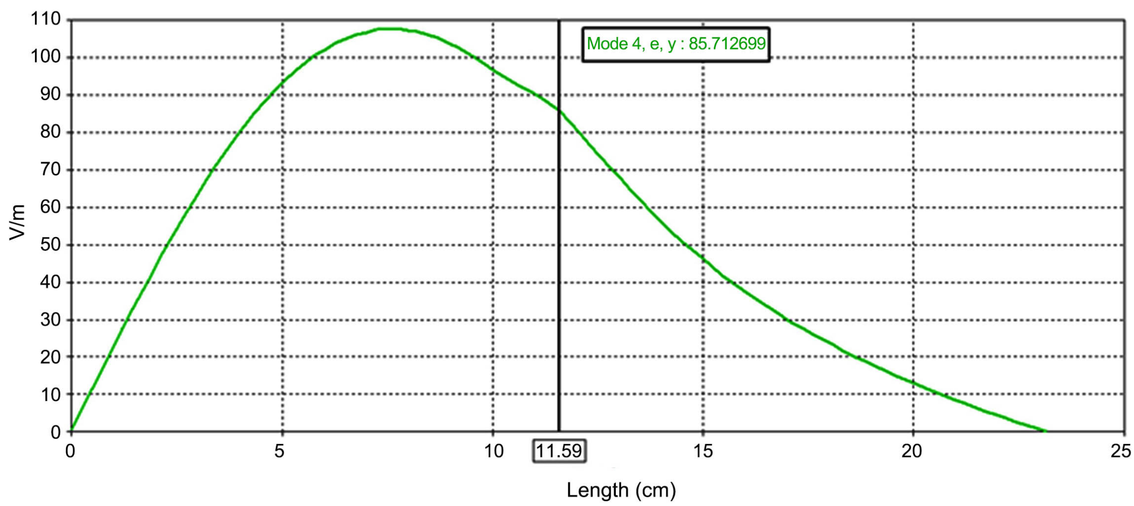


**Figure 5.** The tangential electric field along the radial distribution.

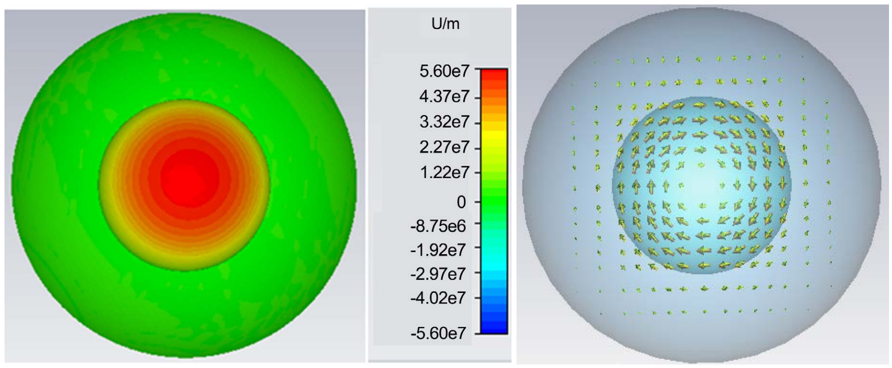


**Figure 6.** The electric field distribution in the composite resonator.

As for the Quality Factor, it is an important parameter for any linear accelerators, which is defined as

$$Q = \omega_0 \frac{U}{P} = 2\pi \frac{U}{T_0 P} \tag{9}$$

In which $\omega_0$ is the resonance angular frequency of the ideal cavity, in which cavity $\sigma = \infty$, $\epsilon'' = 0$, $T_0$ is the corresponding resonance period, $P$ is time averaged cavity power loss, and $U$ is the time energy stored in the cavity

$$U = \frac{1}{16\pi} \int_V \left( \epsilon' |\boldsymbol{E}|^2 + |\boldsymbol{H}|^2 \right) \mathrm{d}V \tag{10}$$

In the above expression, $\mu = 1$ is used.

The power loss caused by the skin current in the metallic wall bounded by the surface $S$ is given by the following expression

$$P_{met} = \frac{c^2}{32\pi^2 \sigma \delta} \int_s |\boldsymbol{H}|^2 \, \mathrm{d}s \tag{11}$$

where

$$\delta = \frac{c}{\sqrt{2\pi\sigma\omega_0}} \tag{12}$$

$\sigma$ is conductivity of the wall, $\delta$ is the skin depth, and the normal component of the magnetic field intensity $\boldsymbol{H}$ is vanishing on the surface $S$ (ideal cavity), so

$\boldsymbol{H} = \boldsymbol{H}_t$

The $Q$ factor of the cavity related to the loss in the metallic wall is given by

$$Q_{met} = \omega_0 \frac{U}{P_{met}} \tag{13}$$

Because at resonance, the averaged electric and magnetic energy stored in the cavity are equal, we can obtain

$$Q_{met} = \frac{2}{\delta} \frac{\int_V |\boldsymbol{B}|^2 \, \mathrm{d}v}{\int_s |\boldsymbol{B}|^2 \, \mathrm{d}s} \tag{14}$$

For $\mu = 1$, $\boldsymbol{H} = \boldsymbol{B}$.

For traditional cylindrical cavity of radius $R_c$ and height $h$,

$$Q_c = 2.405\sqrt{\frac{2\pi\sigma}{\omega_0}}\frac{1}{1+R_c/h} \tag{15}$$

For the SLAC pill box cavity, the typical values are $R_c = h = d = 4\text{ cm}$, $\sigma = 5.294\times10^{17}\text{ s}^{-1}$,then $Q_c = 1.633\times10^4$ can be obtained. The corresponding values for spherical cavities with ideal dielectric spheres and the same values of $d$ and $\sigma$ reach much larger values, see **Figure 7**.

In the presence of the dielectric sphere, we need to deal with the losses due to an imperfect dielectric specified by $\epsilon'' = \operatorname{Im}\epsilon' \neq 0$, so $\operatorname{Im}k \neq 0$, which means that $\omega = \omega' + i\omega''$ is complex.

For the fields given by (1-7), we can get

$$U(t) = U(t=0)\mathrm{e}^{2\omega'' t}$$

So

$$P_{diel} = -\frac{\mathrm{d}U}{\mathrm{d}t} = -2\omega'' U$$

And the corresponding quality will be

$$Q_{diel} = -\frac{\omega_0}{-2\operatorname{Im}\omega} \tag{16}$$

This value is of the order of $\tan\delta^{-1} = \epsilon'/\epsilon''$, and It is approximately l independent. With the ultra-small losses dielectric in [9], $\epsilon' = 10$, $\epsilon'' = 10^{-6}$, $Q_{diel} = 10^7$.

The total quality factor can be obtained by the relation

$$\frac{1}{Q_s} = \frac{1}{Q_{met}} + \frac{1}{Q_{diel}} \tag{17}$$

Values of $Q_s$ versus $l$ for three spherical cavities with dielectric spheres and are shown in **Figure 7**.

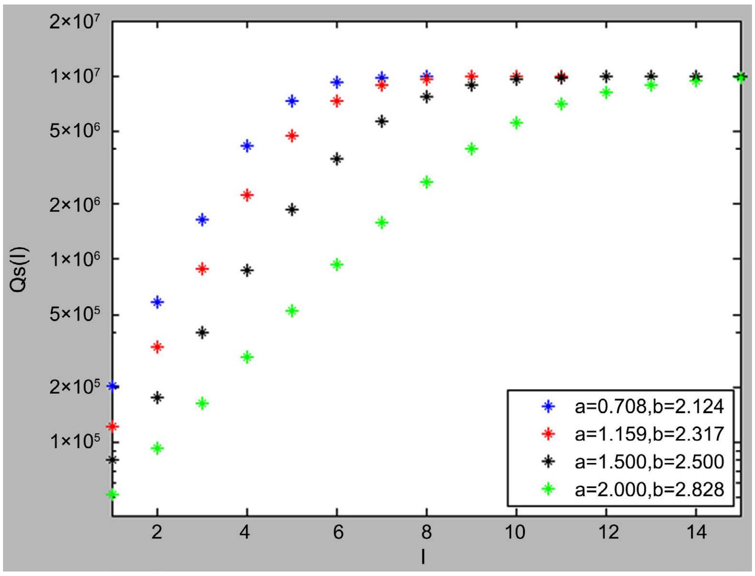


**Figure 7.** The quality vs. $l$ for four spherical cavities with dielectric spheres.

From the calculation results shown in **Figure 7** we can see that, the quality factor in the resonant cavity with a dielectric sphere located at a spherical conducting cavity center are much larger than that in SLAC whose $Q_c = 1.633 \times 10^4$. Furthermore, the quality factor of such a resonator only depends on the losses in the dielectric. For existing ultra-low loss dielectrics, $Q$ can be three orders of magnitude better than obtained in existing cylindrical cavities.

## 3. Conclusion

The above-described details show that, with the use of the composite cavity, we can obtain high accelerating gradient and low loss, especially because all field components at the metallic wall are either zero or very small in this proposed spherical cavity, one can expect the cavity to be less prone to electrical breakdowns than the traditional cavity. It shows potential possibilities of significant increase of acceleration.

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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