Benefit-Function of Two- Identical Cold Standby System subject to Failure due to Natural Frequency Resonance causes catastrophic failures or Failure due to fatigue

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Abstract -Natural Frequency Resonance -Recipe for Failure.

All objects in the universe resonate at their natural frequency when excited. This can cause catastrophic failures in structures, machines, and components. This is relevant in a power plant scenario also. "The principle cannot fail. It is as powerful when applied to the earth as it is when applied to a [violin note shattering a] wineglass, a [boy pushing a man on a] swing, or a steel link. Anyone who doubts should only bear in mind the illustration of the swing. A small boy, by each time adding a pound to the force with which a 200-pound man swings, can soon set the man swinging with the force of 500 pounds. It is necessary only to keep adding a little force at the right time." Nikola Tesla – the master of resonance.

- **Every object depending on its mass and stiffness when excited vibrates at its natural frequency. Watches, musical instruments, microwave ovens, mobile phones and many other devices in our day-to-day life make use of this phenomenon. However, there is an undesirable side to these vibrations that can lead to the failure of structures and components. This failure mode, resonance failures, is equally applicable to large structures and small machine parts alike. Not only bridges, towers and skyscrapers, but also blades, bearings, piping and fasteners can fail due to resonance. Air and gas vapor columns can also resonate at their natural frequencies, in the same way that percussion instruments work, and this can lead to failures.**
- **The war cries and the pounding of foot of marauding armies in history aimed at the destruction of the enemy's defenses by the principle of resonance. The Biblical myth of the Fall of the Walls of Jericho (Joshua 6:3-5) seems to assert this fact.**
- **Large towers and buildings swaying during an earthquake can fail if the swaying coincides with the natural frequency of the structure. Architects consider this while designing these buildings. The Taipei 101, one the tallest building in the world, has a 660 Ton pendulum acting as mass damper to cancel any resonance.**

In this paper we have taken Failure due to Natural Frequency Resonance causes catastrophic failures or fatigue failure. When the main unit fails due to Failure due to Natural Frequency Resonance then cold standby system becomes operative. Failure due to Natural Frequency Resonance cannot occur simultaneously in both the units and after failure the unit undergoes very costly repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

Keywords: Cold Standby, Failure due to Natural Frequency Resonance causes catastrophic failures or fatigue failure, first come first serve, MTSF, Availability, Busy period, Benefit -Function.

INTRODUCTION

Mechanical resonance is the tendency of a mechanical system to respond at greater amplitude when the frequency of its oscillations matches the system's natural frequency of vibration (its resonance frequency or resonant frequency) than it does at other frequencies. It may cause violent swaying motions and even catastrophic failure in improperly constructed structures including bridges, buildings and airplanes—a phenomenon known as **resonance disaster**.

Avoiding resonance disasters is a major concern in every building, tower and bridge construction project. The Taipei 101 building relies on a 660-

ton pendulum — a tuned mass damper — to modify the response at resonance. Furthermore, the structure is designed to resonate at a frequency which does not typically occur. Buildings in seismic zones are often constructed to take into account the oscillating frequencies of expected ground motion. In addition, engineers designing objects having engines must ensure that the mechanical resonant frequencies of the component parts do not match driving vibrational frequencies of the motors or other strongly oscillating parts.

Many resonant objects have more than one resonance frequency. It will vibrate easily at those frequencies, and less so at other frequencies. Many clocks keep time by mechanical resonance in a balance wheel, pendulum, or quartz crystal.

Fatigue failures can be particularly hazardous because they often occur with no visible warning signs and the failure is often sudden and total. While most maintenance technicians understand how to torque fasteners properly, very few understand the reason why it is so critical to torque properly in applications subject to fatigue. Even fewer understand how a fatigue failure really occurs – especially in what they think is a static joint. We often unknowingly avoid fatigue failures in gasketed joints simply because the required crush for the gasket often dictates a torque or bolt tension that minimizes the risk of a fatigue failure. But, at a later time, changing to a new gasket type requiring less crush may set the stage for bolt fatigue failure. Maintenance technicians who don't understand the basic how and why of fatigue can unknowingly set up the conditions for serious failures. Stochastic behavior of systems operating under changing environments has widely been studied. . Dhillon , B.S. and Natesan, J. (1983) studied an outdoor power systems in fluctuating environment . Kan Cheng (1985) has studied reliability analysis of a system in a randomly changing environment. Jinhua Cao (1989) has studied a man machine system operating under changing environment subject to a Markov process with two states. The change in operating conditions viz. fluctuations of voltage, corrosive atmosphere, very low gravity etc. may make a system completely inoperative. Severe environmental conditions can make the actual mission duration longer than the ideal mission duration. In this paper we have taken **Failure due to Natural Frequency Resonance causes catastrophic failures or fatigue failure.** When the main operative unit fails then cold standby system becomes operative. **Failure due to Natural Frequency Resonance causes catastrophic failures** cannot occur simultaneously in both the units and after failure the unit undergoes repair facility of very high cost in case of **Failure due to Natural Frequency Resonance causes catastrophic failures** immediately. The repair is done on the basis of first fail first repaired.

ASSUMPTIONS

1. λ_1 , λ_2, are constant failure rates for **Failure due to Natural Frequency Resonance causes catastrophic failures or fatigue failure.** respectively. The CDF of repair time distribution of Type I and Type II are $G_1(t)$ and $G_2(t)$.
2. The failure due to **Natural Frequency Resonance causes catastrophic failures** is non-instantaneous and it cannot come simultaneously in both the units.
3. The repair starts immediately after the failure due to **Failure due to Natural Frequency Resonance causes catastrophic failures or fatigue failure** works on the principle of first fail first repaired basis.
4. The repair facility does no damage to the units and after repair units are as good as new.
5. The switches are perfect and instantaneous.
6. All random variables are mutually independent.
7. When both the units fail, we give priority to operative unit for repair.
8. Repairs are perfect and failure of a unit is detected immediately and perfectly.
9. The system is down when both the units are non-operative.

NOTATIONS

λ_1, λ_2 are the **Failure due to Natural Frequency Resonance causes catastrophic failures, fatigue failure** respectively. $G_1(t)$, $G_2(t)$ – repair time distribution Type -I, Type-II due to **Failure due to Natural Frequency Resonance causes catastrophic failures, fatigue failure** respectively.

p, q - probability of **Failure due to Natural Frequency Resonance causes catastrophic failures, fatigue failure** respectively such that p+ q=1

$M_i(t)$ System having started from state i is up at time t without visiting any other regenerative state

A_i (t) state is up state as instant t

R_i (t) System having started from state i is busy for repair at time t without visiting any other regenerative state.

B_i (t) the server is busy for repair at time t.

$H_i(t)$ Expected number of visits by the server for repairing given that the system initially starts from regenerative state i

SYMBOLS FOR STATES OF THE SYSTEM

Superscripts O, CS, NFRF, FF

Operative, Cold Standby, **Failure due to Natural Frequency Resonance causes catastrophic failures, fatigue failure** respectively

Subscripts nfrf, frf, ff, ur, wr, uR

No Failure due to Natural Frequency Resonance causes catastrophic failures**;** Failure due to Natural Frequency Resonance causes catastrophic failures, fatigue failure, under repair, waiting for repair, under repair continued from previous state respectively

Up states – 0, 1, 2, 7, and 8;

Down states – 3, 4, 5, 6

regeneration point – 0,1,2, 7, 8

States of the System

0(O_{nnfrf}, CS_{nfrf})

One unit is operative and the other unit is cold standby and there is no failure due to Failure due to Natural Frequency Resonance causes catastrophic failures in both the units.

1($NFRF_{frf, ur}$, O_{nfrf})

The operating unit fails due to Failure due to Natural Frequency Resonance causes catastrophic failures and is under repair immediately of Type- I and standby unit starts operating with no Failure due to Natural Frequency Resonance causes catastrophic failures

2($FF_{ff, ur}$, O_{nfrf})

The operative unit fails due to FF resulting from failure due to fatigue failure and undergoes repair of very costly Type I and the standby unit becomes operative with no Failure due to Natural Frequency Resonance causes catastrophic failures .

3($NFRF_{frf,uR}$, $FF_{ff,wr}$)

The first unit fails due to Failure due to Natural Frequency Resonance causes catastrophic failures and under very costly Type-I repair is continued from state 1 and the other unit fails due to FF resulting from failure due to fatigue failure and is waiting for repair of Type - II.

4($NFRF_{frf,uR}$, $NFRF_{frf,wr}$)

The repair of the unit is failed due to NFRF resulting from failure due to Failure due to Natural Frequency Resonance causes catastrophic failures is continued from state 1and the other unit failed due to NFRF resulting from failure due to Failure due to Natural Frequency Resonance causes catastrophic failures is waiting for repair of very costly Type-I.

5($FF_{ff, uR}$, $FF_{ff, wr}$)

The operating unit fails due to failure due to fatigue failure (FF mode) and under repair of Type - II continue from the state 2 and the other unit fails also due to failure due to fatigue failure is waiting for repair of Type- II.

6($FF_{ff,uR}$, $NFRF_{frf,wr}$)

The operative unit fails due to FF resulting from failure due to fatigue failure and under repair continues from state 2 of Type –II and the other unit is failed due to NFRF resulting from failure due to Failure due to Natural Frequency Resonance causes catastrophic failures and under very costly Type-I

7(O_{nfrf} , $NFRF_{frf,ur}$)

The repair of the unit failed due to NFRF resulting from failure due to Failure due to Natural Frequency Resonance causes catastrophic failures failure is completed and there is no failure due to fatigue failure and the other unit is failed due to NFRF resulting from failure due to Failure due to Natural Frequency Resonance causes catastrophic failures is under repair of very costly Type-I

8(O_{nfrf} , $FF_{ff,ur}$)

The repair of the unit failed due to NFRF resulting from failure due to Failure due to Natural Frequency Resonance causes catastrophic failures failure is completed and there is no failure due to fatigue failure and the other unit is failed due to FF resulting from failure due to fatigue failure is under repair of Type-II.

TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following expressions:

$p_{01} = p, \quad p_{02} = q,$

$p_{10} = pG_1^*(\lambda_1)+q\,G_1^*(\lambda_2) = p_{70},$

$p_{20} = pG_2^*(\lambda_1)+q\,G_2^*(\lambda_2) = p_{80},$

$p_{11}^{(3)} = p(1-G_1^*(\lambda_1)) = p_{14} = p_{71}^{(4)}\; p_{28}^{(5)} = q(1-G_2^*(\lambda_2)) = p_{25} = p_{82}^{(5)} \qquad (1)$

We can easily verify that

$p_{01} + p_{02} = 1, \; p_{10} + p_{17}^{(4)} (=p_{14}) + p_{18}^{(3)} (=p_{13}) = 1,$

$p_{80} + p_{82}^{(5)} + p_{87}^{(6)} = 1 \qquad (2)$

And mean sojourn time is

$\mu_0 = E(T) = \int_0^\infty P[T > t]\,dt$

Mean Time To System Failure

$\text{Ø}_0(t) = Q_{01}(t)[s]\, \text{Ø}_1(t) + Q_{02}(t)[s]\, \text{Ø}_2(t)$

$\text{Ø}_1(t) = Q_{10}(t)[s]\, \text{Ø}_0(t) + Q_{13}(t) + Q_{14}(t)$

$\text{Ø}_2(t) = Q_{20}(t)[s]\, \text{Ø}_0(t) + Q_{25}(t) + Q_{26}(t) \quad (3\text{-}5)$

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-5) and solving for

$$ø_0^*(s) = N_1(s) \,/\, D_1(s) \qquad (6)$$

where

$N_1(s) = Q_{01}^*[\, Q_{13}^*(s) + Q_{14}^*(s)\,] + Q_{02}^*[\, Q_{25}^*(s) + Q_{26}^*(s)\,]$

$D_1(s) = 1 - Q_{01}^* Q_{10}^* - Q_{02}^* Q_{20}^*$

Making use of relations (1) & (2) it can be shown that $ø_0^*(0) = 1$, which implies that $ø_0(t)$ is a proper distribution.

$$\text{MTSF} = E[T] = \left. \frac{d}{ds}\, ø_0^*(s) \right|_{s=0}$$

$$= (D_1'(0) - N_1'(0)) \, / \, D_1(0)$$

$$= (\mu_0 + p_{01}\, \mu_1 + p_{02}\, \mu_2) / (1 - p_{01} p_{10} - p_{02} p_{20})$$

where

$\mu_0 = \mu_{01} + \mu_{02}$,

$\mu_1 = \mu_{01} + \mu_{17}^{(4)} + \mu_{18}^{(3)}$,

$\mu_2 = \mu_{02} + \mu_{27}^{(6)} + \mu_{28}^{(5)}$

AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have

$M_0(t) = e^{-\lambda_1 t} e^{-\lambda_2 t}$,

$M_1(t) = p\, \overline{G_1(t)}\, e^{-(\lambda_1 + \lambda_2)} = M_7(t)$

$M_2(t) = q\, \overline{G_2(t)}\, e^{-(\lambda_1 + \lambda_2)} = M_8(t)$

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t)$

$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) +$
$q_{18}^{(3)}(t)[c]A_8(t) + q_{17}^{(4)}(t)[c]A_7(t)$

$A_2(t) = M_2(t) + q_{20}(t)[c]A_0(t) +$
$[q_{28}^{(5)}(t)[c]\, A_8(t) + q_{27}^{(6)}(t)]\, [c]A_7(t)$

$A_7(t) = M_7(t) + q_{70}(t)[c]A_0(t) +$
$[q_{71}^{(4)}(t)[c]\, A_1(t) + q_{78}^{(3)}(t)]\, [c]A_8(t) \quad A_8(t) = M_8(t) + q_{80}(t)[c]A_0(t)$
$+[q_{82}^{(5)}(t)[c]\, A_2(t) + q_{87}^{(6)}(t)]\, [c]A_7(t) \quad (7\text{-}11)$

Taking Laplace Transform of eq. (7-11) and solving for $\hat{A}_0(s)$

$$\hat{A}_0(s) = N_2(s) / D_2(s) \quad (12)$$

where

$N_2(s) = \widehat{M}_0 (1 - \hat{q}_{78}^{(3)} - \hat{q}_{87}^{(6)}) - \hat{q}_{82}^{(5)}$
$(\hat{q}_{27}^{(6)}\, \hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)} - \hat{q}_{71}^{(4)}$
$(\hat{q}_{17}^{(4)} + \hat{q}_{87}^{(6)}\, \hat{q}_{18}^{(3)}) + \hat{q}_{71}^{(4)}\, \hat{q}_{82}^{(5)}$
$(\hat{q}_{17}^{(4)} - \hat{q}_{27}^{(6)}\, \hat{q}_{18}^{(3)})] + \hat{q}_{01}[\, \widehat{M}_1(1 -$
$\hat{q}_{78}^{(3)}\, \hat{q}_{87}^{(6)}) + \hat{q}_{71}^{(4)}(\widehat{M}_7 + \hat{q}_{78}^{(3)}$
$\widehat{M}_8) + \hat{q}_{18}^{(3)}(\widehat{M}_7\, \hat{q}_{87}^{(6)} - \widehat{M}_8) -$
$\hat{q}_{82}^{(5)}(\widehat{M}_1(\hat{q}_{27}^{(6)}\, \hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) +$
$\hat{q}_{17}^{(4)} (-\widehat{M}_2(\hat{q}_{78}^{(3)} + \widehat{M}_7 \hat{q}_{28}^{(5)}) -$
$\hat{q}_{18}^{(3)}(\widehat{M}_2 + \widehat{M}_7 \hat{q}_{27}^{(6)})\}]\ \hat{q}_{02}[\, \widehat{M}_2(1 - \hat{q}_{78}^{(3)}\ \hat{q}_{87}^{(6)}) + \hat{q}_{27}^{(6)}($
$\widehat{M}_7 + \hat{q}_{78}^{(3)}\, \widehat{M}_8) + \hat{q}_{28}^{(5)}(\widehat{M}_7\, \hat{q}_{87}^{(6)} + \widehat{M}_8) - \hat{q}_{71}^{(4)}(\widehat{M}_1(- \hat{q}_{27}^{(6)} - \hat{q}_{28}^{(5)} +$
$\hat{q}_{87}^{(6)}) + \hat{q}_{17}^{(4)} (\widehat{M}_2 + \hat{q}_{28}^{(5)}\, \widehat{M}_8) - \hat{q}_{18}^{(3)} (-\widehat{M}_2\, \hat{q}_{87}^{(6)} + \widehat{M}_8\, \hat{q}_{27}^{(6)})\}]$
$\hat{q}_{18}^{(3)}(\widehat{M}_2 + \widehat{M}_7 \hat{q}_{27}^{(6)})\}]$

$D_2(s) = (1 - \hat{q}_{78}^{(3)} - \hat{q}_{87}^{(6)}) - \hat{q}_{82}^{(5)}($
$\hat{q}_{27}^{(6)}\, \hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) - \hat{q}_{71}^{(4)}$
$(\hat{q}_{17}^{(4)} + \hat{q}_{87}^{(6)}\, \hat{q}_{18}^{(3)}) + \hat{q}_{71}^{(4)}\, \hat{q}_{82}^{(5)} (\hat{q}_{17}^{(4)}\, \hat{q}_{28}^{(5)} - \hat{q}_{18}^{(3)})] + \hat{q}_{01}[-\hat{q}_{10} (1 -$
$\hat{q}_{78}^{(3)}\, \hat{q}_{87}^{(6)}) - \hat{q}_{71}^{(4)}(\hat{q}_{70} + \hat{q}_{78}^{(3)}$
$\hat{q}_{80}) - \hat{q}_{18}^{(3)}(\hat{q}_{70}\, \hat{q}_{87}^{(6)} - \hat{q}_{80}) -$
$\hat{q}_{82}^{(5)}(-\hat{q}_{10}(\hat{q}_{27}^{(6)}\, \hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) +$
$\hat{q}_{17}^{(4)} (\hat{q}_{20} (\hat{q}_{78}^{(3)} - \hat{q}_{70} \hat{q}_{28}^{(5)}) +$
$\hat{q}_{18}^{(3)}(\hat{q}_{20} + \hat{q}_{70} \hat{q}_{27}^{(6)})\}]\ \hat{q}_{02}[-\hat{q}_{20}(1 - \hat{q}_{78}^{(3)}\ \hat{q}_{87}^{(6)}) - \hat{q}_{27}^{(6)}(\hat{q}_{70} + \hat{q}_{78}^{(3)}\, \hat{q}_{80}) - \hat{q}_{28}^{(5)}(\hat{q}_{70} \hat{q}_{87}^{(6)} + \hat{q}_{80}) - \hat{q}_{71}^{(4)}($
$\hat{q}_{10} (\hat{q}_{27}^{(6)} + \hat{q}_{28}^{(5)}\, \hat{q}_{87}^{(6)}) - \hat{q}_{17}^{(4)} (\hat{q}_{20} -$
$\hat{q}_{28}^{(5)}\, \hat{q}_{80}) - \hat{q}_{18}^{(3)} (\hat{q}_{20}\, \hat{q}_{87}^{(6)} + \hat{q}_{80}$
$\hat{q}_{27}^{(6)})\}]$

(Omitting the arguments s for brevity)

The steady state availability

$$A_0 = \lim_{t\to\infty} [A_0(t)]$$

$$= \lim_{s\to 0} [s\, \hat{A}_0(s)] = \lim_{s\to 0} \frac{s\, N_2(s)}{D_2(s)}$$

Using L' Hospitals rule, we get

$$A_0 = \lim_{s\to 0} \frac{N_2(s) + s\, N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)} \quad (13)$$

The expected up time of the system in (0,t] is

$$\lambda_u(t) = \int_0^{\infty} A_0(z)\, dz$$

So that $\overline{\lambda_u}(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)}$ (14)

The expected down time of the system in (0,t] is

$\lambda_d(t) = t - \lambda_u(t)$

So that $\overline{\lambda_d}(s) = \frac{1}{s^2} - \overline{\lambda_u}(s)$ (15)

The expected busy period of the server when there is FF - Failure due to fatigue failure or NFRF- failure due to Natural Frequency Resonance causes catastrophic failures in (0,t]

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t)$

$R_1(t) = S_1(t) + q_{10}(t)[c]R_0(t) +$

$q_{18}^{(3)}(t)[c]R_8(t) + q_{17}^{(4)}(t)[c]R_7(t)$

$R_2(t) = S_2(t) + q_{20}(t)[c]R_0(t) + q_{28}^{(5)}(t)$

$R_8(t) + q_{27}^{(6)}(t)][c]R_7(t)$

$R_7(t) = S_7(t) + q_{70}(t)[c]R_0(t) + Q_{71}^{(4)}(t)$

$R_1(t) + q_{78}^{(3)}(t)][c]R_8(t)$

$R_8(t) = S_8(t) + q_{80}(t)[c]R_0(t) + Q_{82}^{(5)}(t)$

$R_2(t) + q_{87}^{(6)}(t)][c]R_7(t)$ (16-20)

Taking Laplace Transform of eq. (16-20) and solving for $\overline{R_0}(s)$

$\overline{R_0}(s) = N_3(s) / D_2(s)$ (21)

where

$N_3(s) = \hat{q}_{01}[\hat{S}_1(1 - \hat{q}_{78}^{(3)}\hat{q}_{87}^{(6)}) +$

$\hat{q}_{71}^{(4)}(\hat{S}_7 + \hat{q}_{78}^{(3)}\hat{S}_8) + \hat{q}_{18}^{(3)}(\hat{S}_7$

$\hat{q}_{87}^{(6)} - \hat{S}_8)] - \hat{q}_{01}\hat{q}_{82}^{(5)}(\hat{S}_1\hat{q}_{27}^{(6)}\hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) + \hat{q}_{17}^{(4)}$
$(\hat{S}_2\hat{q}_{78}^{(3)} + \hat{S}_7\hat{q}_{28}^{(5)}) -$

$\hat{q}_{18}^{(3)}(\hat{S}_2 + \hat{S}_7\hat{q}_{27}^{(6)})] + \hat{q}_{02}[\hat{S}_2(1 -$

$\hat{q}_{78}^{(3)}\hat{q}_{87}^{(6)}) + \hat{q}_{27}^{(6)}(\hat{S}_7 + \hat{q}_{78}^{(3)}\hat{S}_8) + \hat{q}_{28}^{(5)}(\hat{S}_7\hat{q}$
$_{87}^{(6)} + \hat{S}_8) - \hat{q}_{02}\hat{q}_{71}^{(4)}(\hat{S}_1(-$

$\hat{q}_{27}^{(6)} - \hat{q}_{28}^{(5)}\hat{q}_{87}^{(6)}\hat{q}_{17}^{(4)}(\hat{S}_2 + \hat{q}_{28}^{(5)}\hat{S}_8) - \hat{q}_{18}^{(3)}(-\hat{S}_2\hat{q}$
$_{87}^{(6)} + \hat{S}_8\hat{q}_{27}^{(6)})]$

and

$D_2(s)$ is already defined.

(Omitting the arguments s for brevity)

In the long run, $R_0 = \frac{N_3(0)}{D_2'(0)}$ (22)

The expected period of the system under FF - failure due to fatigue failure or NFRF- Failure due to Failure due to Natural Frequency Resonance causes catastrophic failures in (0,t] is

$\lambda_{rv}(t) = \int_0^{\infty} R_0(z)dz$ So that $\overline{\lambda_{rv}}(s) = \frac{\hat{R}_0(s)}{s}$

The expected number of visits by the repairman for repairing the identical units in (0,t]

$H_0(t) = Q_{01}(t)[s][1 + H_1(t)] +$

$Q_{02}(t)[s][1 + H_2(t)]$

$H_1(t) = Q_{10}(t)[s]H_0(t)] + Q_{18}^{(3)}(t)[s]$

$H_8(t) + Q_{17}^{(4)}(t)][s]H_7(t)$,

$H_2(t) = Q_{20}(t)[s]H_0(t) + Q_{28}^{(5)}(t)[s]$

$H_8(t) + Q_{27}^{(6)}(t)][c]H_7(t)$

$H_7(t) = Q_{70}(t)[s]H_0(t) + Q_{71}^{(4)}(t)[s]$

$H_1(t) + Q_{78}^{(3)}(t)][c]H_8(t)$

$H_8(t) = Q_{80}(t)[s]H_0(t) + Q_{82}^{(5)}(t)[s]$

$H_2(t) + Q_{87}^{(6)}(t)][c]H_7(t)$ (23-27)

Taking Laplace Transform of eq. (23-27) and solving for $H_0^*(s)$

$H_0^*(s) = N_4(s) / D_3(s)$ (28)

In the long run, $H_0 = N_4(0) / D_3'(0)$ (29)

BENEFIT- FUNCTION ANALYSIS

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure due to Failure due to Natural Frequency Resonance causes catastrophic failures or failure due to fatigue failure, expected number of visits by the repairman for unit failure.

The expected total Benefit-Function incurred in (0,t] is

C(t) = Expected total revenue in (0,t]

- expected busy period of the system under failure due to Failure due to Natural Frequency Resonance causes catastrophic failures or failure due to fatigue failure for repairing the units in (0,t]

- expected number of visits by the repairman for repairing of identical the units in (0,t]

The expected total cost per unit time in steady state is

$C = \lim_{t\to\infty}(C(t)/t) = \lim_{s\to 0}(s^2 C(s))$

$= K_1A_0 - K_2R_0 - K_3H_0$

where

K_1 - revenue per unit up-time,

K_2 - cost per unit time for which the system is under repair of type- I or type- II

K_3 - cost per visit by the repairman for units repair.

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to Failure due to Natural Frequency Resonance causes catastrophic failures or failure due to fatigue failure increases, the

MTSF and steady state availability decreases and the Benefit-function decreased as the failure increases.

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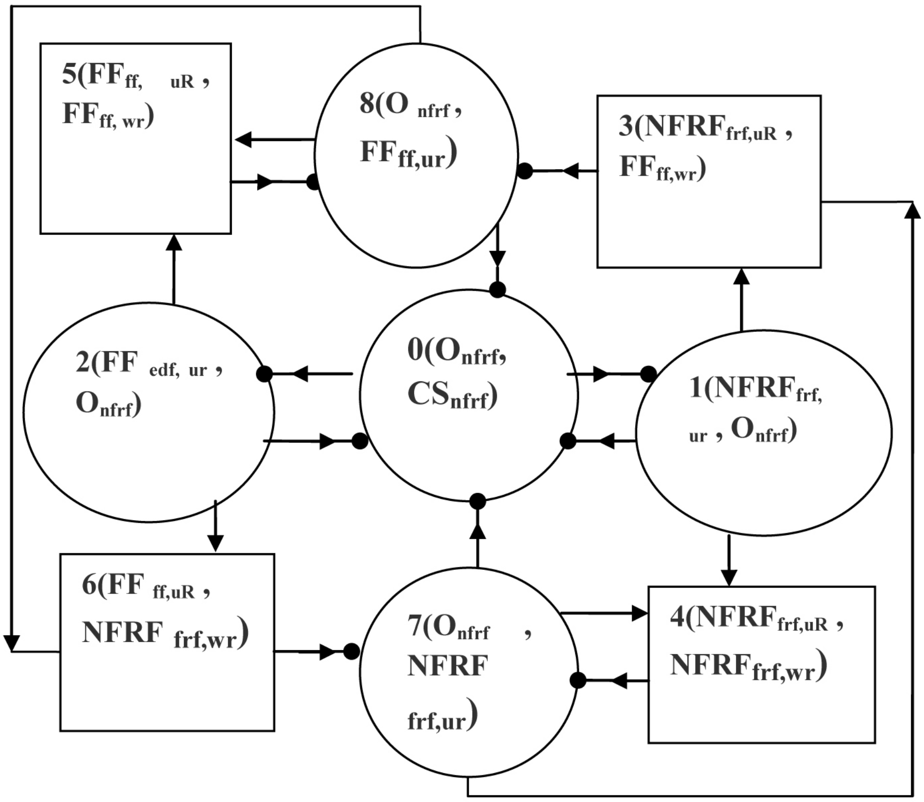

Fig. The State Space Diagram

○ Up state □ down state

• Regeration point