acoustics

Review

A Review of Finite Element Studies in String Musical Instruments

Evaggelos Kaselouris [1,2], Makis Bakarezos [1,2], Michael Tatarakis [2,3], Nektarios A. Papadogiannis [1,2] and Vasilis Dimitriou [1,2,*]

[1] Physical Acoustics and Optoacoustics Laboratory, Department of Music Technology & Acoustics, Hellenic Mediterranean University, 74100 Rethymnon, Greece; vagfem@hmu.gr (E.K.); bakarezos@hmu.gr (M.B.); npapadogiannis@hmu.gr (N.A.P.)
[2] Institute of Plasma Physics & Lasers—IPPL, Hellenic Mediterranean University Research Centre, 74100 Rethymnon, Greece; mictat@hmu.gr
[3] Department of Electronic Engineering, Hellenic Mediterranean University, 73133 Chania, Greece
* Correspondence: dimvasi@hmu.gr

Abstract: String instruments are complex mechanical vibrating systems, in terms of both structure and fluid–structure interaction. Here, a review study of the modeling and simulation of stringed musical instruments via the finite element method (FEM) is presented. The paper is focused on the methods capable of simulating (I) the soundboard behavior in bowed, plucked and hammered string musical instruments; (II) the assembled musical instrument box behavior in bowed and plucked instruments; (III) the fluid–structure interaction of assembled musical instruments; and (IV) the interaction of a musical instrument's resonance box with the surrounding air. Due to the complexity and the high computational demands, a numerical model including all the parts and the full geometry of the instrument resonance box, the fluid–structure interaction and the interaction with the surrounding air has not yet been simulated.

Keywords: finite element method; string musical instruments; modeling and simulation

check for updates

Citation: Kaselouris, E.; Bakarezos, M.; Tatarakis, M.; Papadogiannis, N.A.; Dimitriou, V. A Review of Finite Element Studies in String Musical Instruments. *Acoustics* **2022**, *4*, 183–202. https://doi.org/10.3390/acoustics4010012

Academic Editors: Jian Kang and Li Cheng

Received: 26 November 2021
Accepted: 21 February 2022
Published: 23 February 2022

1. Introduction

In musical acoustics, musical instruments are categorized based on criteria that depend on the excitation mechanisms. These mechanisms have an important influence on the spectra of the sounds. Thus, in terms of drive mechanisms, musical instruments of the symphony orchestra in the Western European tradition are classified into string instruments, wind instruments and percussion instruments [1]. This review is focused on the stringed instruments which produce sound from vibrating strings when a performer plays the strings in some manner. A stringed instrument consists of a structure with a cavity which holds strings under tension. The structure radiates sounds if the strings vibrate. When the strings vibrate, they produce a time-dependent force that sets the soundboard (top plate) into motion. String instruments are complex mechanical vibrating systems, in terms of both structure and the fluid–structure interaction [2]. The vibration of the air cavity and the main body of the instrument is the main source of the produced sound, while the string excitation alone has a negligible role in the produced sound. The whole instrument acts as a filter that converts the excitation force of the strings to radiated sound [3].

The stringed instruments can be excited by bowing, plucking or hammering [4,5]. Representative bowed instruments are the violin, viola, cello, double bass, Yaylı tambur, rebab and Cretan lyra. The plucked instruments are divided into short-necked plucked instruments such as guitar, lute, oud, pipa and mandolin and long-necked plucked instruments such as baglama, bouzouki, veena, setar and theorbo. The harp and the harpsichord are built without a neck, and their plucking mechanism differs from the aforementioned instruments; therefore, although they are plucked instruments, they do not belong to these two categories. The piano and the dulcimer are representative hammered instruments.

According to instrument builders and musicians, the four important features which determine the instrument quality and character are the timbre (sound quality), the attack behavior, the overall loudness and the degree of the possible timbre variation [6]. The importance of timbre is evident mainly within a specific instrument family. Musicians judge instruments to be interesting when they show a wide variety of possible timbres and articulation changes [7]. Control of the attack behavior is also critical since instruments with fast attack allow for fast playing [7]. The maximization of the instrument loudness is also essential for instrument builders. Indicatively, an important element that contributes decisively to the radiation efficiency of the violin is the existence of the sound post. Furthermore, the degree of possible timbre variation produced by the player depends on the response of the instrument to loudness variations [6].

The physical behavior of a stringed instrument can be examined under three considerations: firstly, the stretched string behavior, whose vibration is controlled by the player; secondly, the response of the soundbox of the instrument and the neighboring air in response to the string motion; and thirdly, the radiated sound, almost entirely from the soundbox. All three are interconnected and should not be treated independently [8]. In many of the string instruments, the mechanical vibrations are transmitted to the body of the instrument, which usually incorporates a hollow or encloses a volume of air. The vibration of the body of the instrument and the enclosed air or chamber enables the vibration of the string to be more audible to the performer and audience. For the majority of the string instruments, the body is hollow; however, instruments that rely on electronic amplification, such as the electric guitar, may have a solid body. The air cavity of a string instrument, such as the violin or the acoustic guitar, functions as a Helmholtz-type resonator that reinforces certain frequencies [3].

This review paper is structured in eight sections. In Section 2, the basic concepts of the modal analysis of musical instruments are presented. Section 3 introduces the numerical methods used to simulate the vibrational behavior of musical instruments. An overview of the FEM simulation studies focused on the soundboards of stringed instruments is presented in Section 4. In Section 5, literature studies on the vibration analysis of the assembled instrument box are presented, while fluid–structure interaction studies that consider the enclosed air of the assembled instrument box are presented in Section 6. Section 7 introduces studies of the stringed instruments that produce sound vibrations interacting with the surrounding air. The conclusions of this review are summarized in Section 8.

2. Modal Analysis

The complex vibrations of musical instruments can be described in terms of normal modes of vibration. Modal analysis of musical instruments is the study of their dynamic properties under vibrational excitation. For this case, the instrument can be viewed as an elastic structure. The frequency response of a musical instrument is found when one sums the modal responses of its substructures according to their degree of participation in the structural motion. Each mode of vibration is identified by three main parameters, namely the natural frequency, the mode shape and the damping factor. Any deformation pattern of a musical instrument is expressed by a combination of the mode shapes. The mode shape is a deflection pattern that is related to a specific natural frequency and represents the relative displacement of all parts of the instrument, in various directions, for that mode. The damping factor of each mode is coupled to its natural frequency, while it is inversely proportional to the mass distribution [4,9].

Experimental modal testing allows identification of modal parameters of vibrating musical instruments such as natural frequencies, mode shapes and modal damping for the substructures of the musical instruments. Modal testing may use continuous (sinusoidal), impulsive or random excitation and may measure the response mechanically, optically or indirectly by observing the radiated sound field. Mechanical techniques require an excitation device such as a roving hammer or a fixed automated force hammer impacting the bridge of the stringed instrument [4]. Optical methods for vibration measurements, such as holographic interferometry [10], laser Doppler vibrometry [11] and laser Doppler velocimetry [12], are nondestructive, while holographic interferometry techniques [10,13] have the advantage of full-field imaging. Electronic speckle pattern interferometry (ESPI), also known as TV holography, is a technique which uses laser light together with video detection recording and processing to visualize static and dynamic displacements of components with optically rough surfaces [13,14]. The excitation force is measured with a force transducer (piezoelectric transducer or load cell), the acceleration is measured with an accelerometer and the structure velocity response is measured with a laser velocimeter or by holographic interferometry. Data are converted into digital signals and stored on a host computer. The analysis of the experimental signals is mainly performed with Fourier analysis. The resulting frequency response function (FRF) demonstrates characteristic peak resonances for different frequencies. Depending on the problem needs, the FRF is expressed by several parameters, such as compliance (displacement/force), mobility (velocity/force), accelerance (acceleration/force), dynamic stiffness (compliance^{-1}), impedance (mobility^{-1}) and dynamic mass (accelerance^{-1}) [4].

The influence of many variables such as thickness, curvature, material properties, density and elastic constants on the instrument's vibration modes can be demonstrated via modal analysis numerical simulations that provide significant insights to experimental modal testing. Mathematical modal analysis is mainly performed via the finite element method (FEM) and the finite difference method (FDM). Representative works which address the fundamentals of the modeling and simulation of continuous vibrating elastic systems by the FEM can be found in [15,16].

3. Numerical Methods

Numerical methods are capable of simulating complicated processes, i.e., when the material properties are anisotropic, viscoelastic or temperature-dependent, as well as complicated geometries of dynamic structures, such as the geometries of string musical instruments. The basic numerical methods used are the FEM, FDM and boundary element method (BEM). The numerical methods have many advantages compared to the analytical methods since the solution domain is divided into many smaller domains that are allowed to have different values of physical properties and/or varying loading conditions.

The finite element analysis is based on approximate solutions to systems of partial differential equations [17,18]. The modeling analysis divides the behavior of complex structures into small elements. The FEM is a series of computerized numerical calculations based on matrices. It is versatile due to its flexibility in modeling complicated geometries when the domain changes, when the desired precision varies over the entire domain or when the solution lacks smoothness [19,20]. During FEM preprocessing, the structure is subdivided into a mesh (or grid) that, depending on the geometry, may consist of one-dimensional line elements; two-dimensional area elements, such as triangles or rectangles; or three-dimensional volume elements, such as regular or irregular tetrahedra or hexahedra or other polyhedra. The solution is approximated using a collection of the so-called shape functions, which may or may not lie in a regular arrangement. Moreover, the numerical solution is usually approached using integral and variational methods. In addition, the mechanical properties such as the density and elastic constants including Young's modulus, Poisson's ratio and shear modulus are necessary, and even thermal properties such as thermal conductivity and heat capacity may be needed, along with proper boundary conditions.

The finite difference method (FDM) is capable of solving problems defined either in one spatial dimension or over a simple geometry in two dimensions. This concerns all string models, 1D tube models, bars and rectangular or circular percussion instruments. In general, finite difference methods are used for regular grids. In FDM, the structure is not divided into a mesh of elements, but rather discrete node points are identified on the geometry and the equation system is solved for these points, considering differences with the neighboring points [21–23]. Therefore, the FEM formulation is more complicated and demands more computational time. Its advantage is the greater ease of handling boundaries. Moreover, the selection of the element geometry and element density in FEM is very flexible, making it suitable for complicated geometries. Bader [24,25] successfully performed 3D FDM simulations of the complete geometry of the classical guitar.

In musical acoustics, the BEM formulation is commonly used to calculate the sound radiated by simulated musical instruments [26–28]. The BEM is a numerical computational method capable of solving linear partial differential equations, which have been formulated as integral equations. The integral equation is regarded as an exact solution of the governing partial differential equation. The boundary element method attempts to use the given boundary conditions to fit boundary values into the integral equation, rather than values throughout the space defined by a partial differential equation. After fitting, the integral equation is used again in the postprocessing stage to calculate numerically the solution directly at any desired point in the interior of the solution domain.

Finite element analysis is ideal for predicting how musical instruments react to any kind of force loads, vibrations and variations in environmental conditions (temperature, relative humidity, etc.). The basic equation of motion solved for an FEM structural dynamic analysis is as follows:

$$[M]\left\{\frac{\partial^2 U}{\partial t^2}\right\} + [C]\left\{\dot{U}\right\} + [K]\{U\} = \{F\} \tag{1}$$

where $[M]$ is the mass matrix, $[C]$ is the damping matrix, $[K]$ is the stiffness matrix, $\{U\}$ is the displacement vector and $\{F\}$ is the load vector. Ignoring damping and external forces, for a harmonic motion in the frequency domain, Equation (1) results in a modal eigenvalue problem of the following form:

$$\left([K] - \omega^2[M]\right)[\Phi] = 0 \tag{2}$$

where $[\Phi]$ is the modal matrix, whose columns are eigenmodes and ω are angular eigenfrequencies.

4. FEM Studies on Soundboards of String Musical Instruments

The motion of soundboards (top plates) is essential in sound production. The soundboards are usually made from spruce wooden material in string musical instruments such as violin, guitar and piano. This selection is justified due to the fact that the ratio of Young's modulus (of the strongest direction, along the grain) to density is very large. Like in most woods, the elastic constants of spruce depend strongly on the direction, and in general, a total of 27 different constants are required to fully describe its elastic behavior [5].

Contemporary instrument making is still based more on tradition than understanding, and a definitive comprehensive scientific study that directly relates the instrument shape with the vibrational properties is still nonexistent. It is worth noticing that the modal frequencies (eigenfrequencies) of the free top plates are not directly related to the acoustic properties of the complete instrument. However, according to instrument makers, the soundboard's modal frequencies are crucial parameters that decisively affect the selections during the construction of the instrument. After identifying the problematic notes on a particular instrument, the luthier can check the general modal map of the instrument and identify the modes that provoke the problems. Moreover, the knowledge of the mode shapes provides a guideline as to which areas of the structure should be targeted in order to fix particular modes with minimal disturbance to other modes/frequencies [29]. This section presents an overview of the FEM simulation studies focused on the soundboards of stringed instruments.

4.1. Bowed Stringed Musical Instruments

The violin is the most well studied bowed musical instrument in musical acoustics [30–32]. In recent years, there has been significant improvement in the knowledge of the violin vibrational modes, which are responsible for the intensity and quality of the instrument's sound. This knowledge is derived from finite element simulations [33–36] and experimental investigations [37–40]. Nevertheless, the physics of the bowed strings and the complex shape and structure of the violin lead to many complications that are still the object of ongoing research. The understanding and modeling of the acoustically important vibrational modes of the violin and related instruments are still less well advanced.

Regarding the soundboards, there are many factors that affect their vibrational behavior, such as the geometrical characteristics (arching, thickness and distribution of thicknesses), the material properties and the anisotropy and inhomogeneity of the materials. In the literature, after the pioneering experimental work of Hutchins [41] there are various studies [42–48] that numerically study the violin free top and back plates, providing valuable guidance to violinmakers on the aforementioned factors that influence the plate frequencies and mode shapes before the final assembly on the instrument. Molin et al. [48] investigated the influence of the overall plate thickness variation, local thickness variations, arching height and material parameters, while experimental measurements of Chladni patterns were used for comparison with the FEM results. Likewise, Bretos et al. [34] studied the tuning of free violin plates and compared the numerical results with the experimental results of Richardson et al. [49].

A notable study is that of Gough [50], who developed different models starting from simple assumptions and then gradually added the complications of violin design. Hence, the progressive evolution of the signature modes was charted, and the relative influence of various contributory factors was evaluated. The various contributory factors that were investigated concerned the influence of arching, elastic anisotropy, plate thickness, the f-holes and the island area, extensional and rotational constraints, the sound post and the bass bar. Representative results of increasing the plate thickness are reprinted from [50] in Figures 1 and 2. Figure 1 demonstrates the computed frequencies of the first 16 modes of a 15 mm arched violin plate, for a wide range of plate thicknesses, plotted on a logarithmic scale. Each color represents a mode, starting from mode #1 to mode #16. The #5/#2 frequency ratio is significant for octave tuning [50]. For the 15 mm arched plate, the computed ratio falls from 2.97 for 1 mm, 2.46 for 2 mm and 2.11 for 3 mm to 1.86 for 4 mm thick plates. The results demonstrate that the average ratio of 2.3 for fine Italian violins [51] may be reproduced by a uniform spruce plate thickness slightly less than 2.5 mm. Furthermore, the violinmakers are also aware of the dependence of the free plate mode frequencies and shapes on selective graduation of plate thickness. Therefore, an example of plate thickness proportional to the arching height rising, from 2 mm at the plate edges to a maximum thickness at the plate center, was also presented. Figure 2 shows the increment of the computed modal frequencies in relation to the midplate thickness.

Recently, Gonzalez et al. [52,53] demonstrated that the modal frequencies of the violin soundboards can be predicted by an artificial intelligence neural network based on their geometric parameters; furthermore, it is observed that this approach can be successfully adopted by traditional violinmakers. Moreover, the studies of Lu [47] and Kaselouris et al. [54] explored the possibility of using composite materials to substitute the traditional wood material of the top plates. The advantage of using composite materials lies in their minimum sensitivity to humidity and temperature changes which makes them almost unaffected by the environment within which they are being played. Additionally, composites are high-strength materials with great machinability in contrast to wood. The results of both of these research studies showed that the vibrational behavior of composite soundboards is significantly different from that of traditional wooden soundboards.

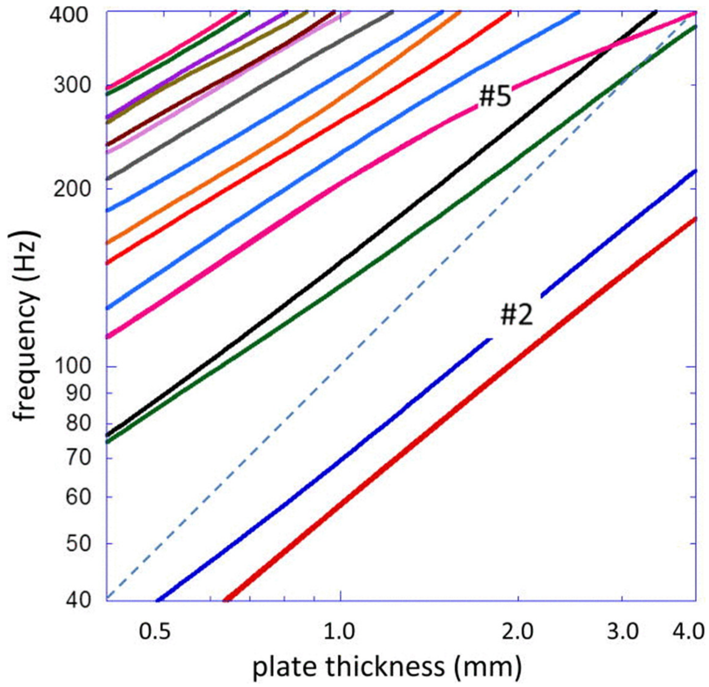

Figure 1. Influence of plate thickness on the free plate modes of a 15 mm arched violin top plate (without f-holes or bass bar) considering a geometric mean Young's modulus. The dashed line indicates the thickness dependence for a flat plate. Reprinted with permission from [50]. Copyright 2015 Acoustic Society of America.

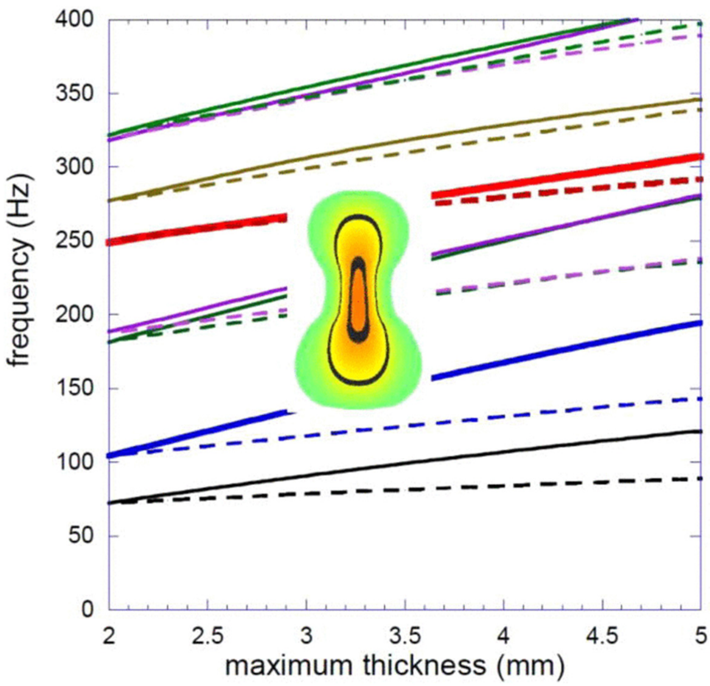

Figure 2. The increase in modal frequencies of a freely supported 15 mm arched violin plate, as the thickness of the plate increases linearly with local arching height, from 2 mm at the outer edges to a maximum thickness at the center of the plate. The dashed lines show the modal frequencies for a plate of the same mass but with uniform thickness. The contours on the inset indicate the thickness at 1/3 and 2/3 maximum. The physical constants are the same as those used for Figure 1. Reprinted with permission from [50]. Copyright 2015 Acoustic Society of America.

Regarding other studies that concern bowed instruments, it is worth noting the work of Wilczyński et al. [55] on a viola da gamba. The main objective of this numerical study was to induce prestresses on the soundboard and to examine the differences between prestressed and non-prestressed models. The main conclusions were that all the frequencies decrease when prestresses exist, while most of the modes are different between models with and without prestresses. Moreover, in the work of Bakarezos et al. [56], the vibration characteristics of a traditional Cretan lyra soundboard were studied via FEM and experimental ESPI measurements. The vibration amplitude distributions, the eigenmodes, obtained by time average ESPI for each eigenfrequency were found to be in good agreement with the computational results of the FEM simulations.

4.2. Plucked Musical Instruments

The guitar, acoustic or electric, is the most popular musical instrument in the world. The classical guitar's sound characteristics depend on the vibratory interaction of the whole instrument with a specific string excitation. The soundboard is a crucial part for this interaction, which to a large extent determines the guitar's musical acoustic quality [5,57]. It was experimentally shown that the thickness and shape of the soundboard as well as the characteristics of the braces, such as their number, shape and orientation, affect the mechanical behavior of a guitar [57–59].

The vibrational behavior of a guitar plate as a function of the mechanical parameters of wood was studied for a meaningful set of samples via the FEM in [60]. It was found that the most influential parameters for the normal frequencies and the modes are the Young's moduli. The values of the computed eigenfrequencies strongly depended on the variation of the transversal and longitudinal Young's moduli. The authors concluded that the determination of the precise values of Young's moduli is crucial to study the vibrational behavior. In [61], the influence of changing the relative humidity of air on the mechanical properties of a guitar's top plate was measured. Vibratory responses of the plate were experimentally measured. To determine the connection between the shift in natural frequencies and the changes in the wood's elastic and shear moduli, an FEM model of the plate was developed, and modal analysis was carried out. The variations of the obtained results due to this wood–air interaction were considered responsible for the modified vibrational behavior of the musical instruments. Shepherd et al. [62] modeled an acoustic guitar soundboard by FEM using orthotropic material properties. The variability of the wood properties was also considered including the effect of the moisture content. The measured modes and the natural frequencies resulted in a good agreement with the experimental measurements. Furthermore, an uncertainty analysis was performed using the normal variance of the wood properties, based on values that have been reported in the literature. The results of this analysis highlighted that the accuracy of the computational results depends on the values of the material properties.

A notable research work of Torres et al. [59] studies the influence of two different bridge configurations on the vibrations of the top plate of a classical guitar via detailed damped simulations using the FEM, experimental harmonic analysis and visualization techniques. The results of this work showed that a simple change in the structural configuration noticeably affects the vibration patterns of the instrument. Such a representative result is here reprinted from [59] in Figure 3. The simulated (top) and experimental (bottom) operational deflection shapes of the top plate velocity without bridge, in its fourth resonance peak excited with 0.035 N, are shown. A good agreement between the numerical and the experimental top plate velocity distributions was found.

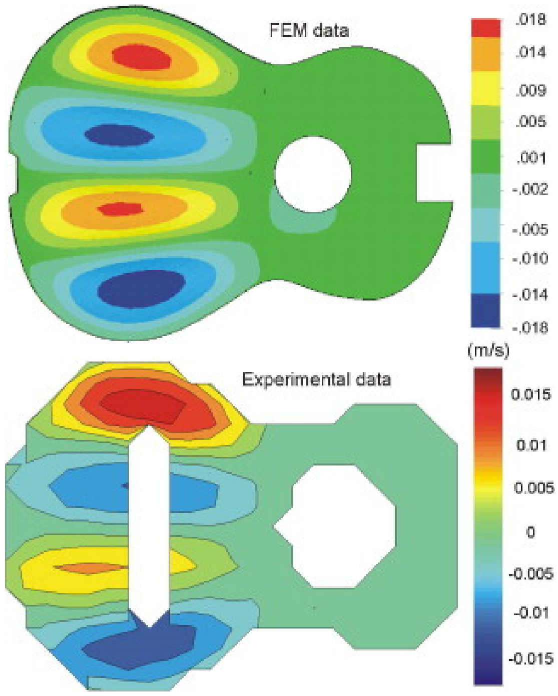

Figure 3. Comparison of simulated (**top**) and experimental (**bottom**) operational deflection shapes of the top plate velocity without bridge in its fourth resonance peak excited with 0.035 N. Reprinted with permission from [59]. Copyright 2009 Elsevier.

Recently, Salvi et al. [63] analyzed the modal response of the free top plates of archtop guitars via FEM and investigated the similarities of its mode shapes with those of similar instruments, such as the violin and the classical guitar. Moreover, Viala et al. [64] demonstrated, via numerical and experimental modal analysis and uncertainty quantification, that the specific elastic parameters of the wood (in longitudinal and radial directions and longitudinal–radial plane) mainly influence the dynamics of the soundboard, whilst the relative humidity changes have a non-negligible impact.

4.3. Hammered Musical Instruments

The strings of the piano, which is the most characteristic hammered musical instrument, are set into motion by a hammer impact. Strings and soundboard are connected through bridges and the soundboard is stiffened via a set of ribs [5]. The piano soundboard transforms the string vibration into sound. These vibrations are essential for the sound characteristics of the instrument. One of the first FEM models of a piano soundboard was developed by Kindel and Wang [65]. The piano structure was modeled via beam elements, while vibration modes up to 130 Hz were compared with modal experiments. Berthaut et al. [66] developed a 2D FE model using shell elements in-plane and performed experimental modal analysis on a grand piano soundboard. The comparison of numerical and experimental measurements demonstrated a satisfactory correlation up to 250 Hz. A simplified FE model of the soundboard by Moore and Zietlow [67], along with ESPI experimental measurements, demonstrated that the vertical force exerted by the strings on the soundboard has a non-negligible effect on the natural frequencies of the lowest modes, while it has negligible effects for mid/high frequencies. An FE model of the piano soundboard was developed by Mamou-Mani et al. [68] to study the effects of the tension of the string (downbearing) on its vibration, considering the ribs, the bridges and the crown. Prestress calculation was considered. Results were presented in the frequency range for mode resonances up to 450 Hz. A simple phenomenological law was derived to describe the evolution of eigenfrequencies with downbearing, including the initial crown. A vibroacoustic study of Ege et al. [69], combining FE analysis and modal analysis, concluded that below 1 kHz, the soundboard vibrates similarly to a homogeneous plate, while above that limit the structural waves are confined by the ribs. Recently, Corradi et al. [70] studied the manufacturing processes of the grand piano soundboard via FEM and modal analysis. Three stages of manufacturing were investigated: (i) freely suspended soundboard prior to the gluing of the bridges, (ii) freely suspended soundboard including the two bridges and (iii) the soundboard attached to the piano frame. In order to pair the numerical and the experimental modes, the modal assurance criterion (MAC) was adopted [70]. Representative numerical and experimental results of stage (ii) are demonstrated in Figure 4. The agreement between the paired numerical and experimental modes, in terms of the natural frequency, the MAC value and the visual representation of the mode shapes, is very satisfactory.

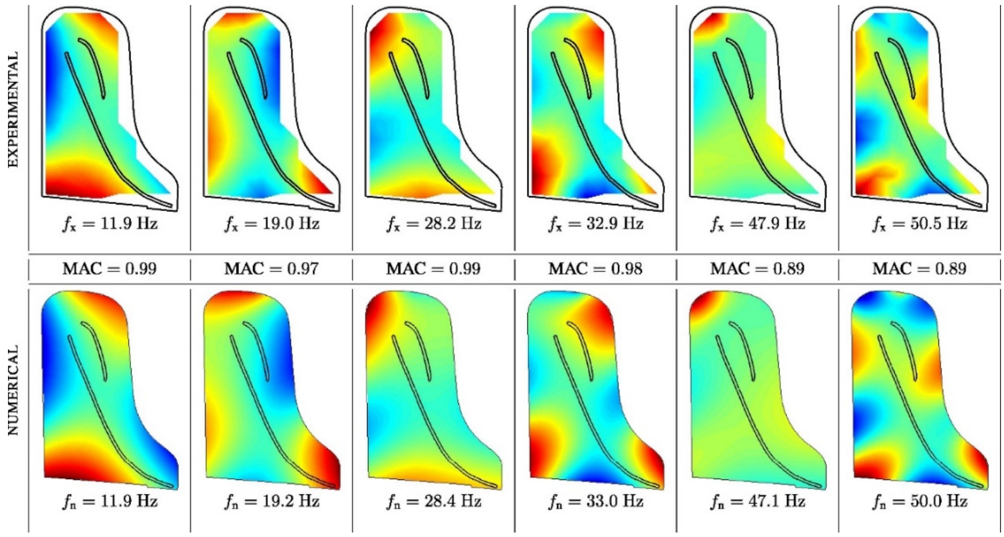

Figure 4. First six experimental vibration modes vs. numerical ones. Simulations include both bridge gluing and rib planning. Reprinted with permission from [70]. Copyright 2017 Elsevier.

5. FEM Studies of Assembled String Musical Instrument Box

The modeling of the whole body of a musical stringed instrument is a more complex process which leads to high computational cost demanding simulations. In addition, the number of the material parameter values increases, and accurate values of these numerous different materials have to be assigned to each individual part of the instrument assembly. Thus, an accurate computation of the derived normal modes demands a very sophisticated modeling. In this section, literature studies on the vibration analysis of the assembled instrument box are presented, while fluid–structure interaction simulations that couple the top plate with the Helmholtz air resonance of the body cavity are presented in Section 6. In Section 7, simulations of the radiated sound with FEM–BEM coupling are demonstrated.

5.1. Bowed Musical Instruments

Preliminary work to study the modal characteristics of a complete violin assemblage with the Stradivari shape, via FEM modeling, was conducted by Knott et al. [33], while the numerical results were validated via an experimental modal analysis [37]. Bretos et al. [34] modeled the whole violin box, except the bridge, the neck, the fingerboard and the strings, and the first 10 eigenmodes were calculated. An effort was made to establish relationships between the vibrational behavior of the free plates and the assembled violin structure. The structural parameters of the violin box, such as the thickness of the violin plate; the height, distribution and change of the shape of the arch in the front plate geometry; and the status of the bass bar and sound post, were investigated by Zhang et al. [71], for their influence on the vibration patterns and frequency response of a violin box. The assembly of the model was not full since the neck, the fingerboard and the strings were not modeled. To verify the results of FEM simulation, experiments to measure the vibration frequency response were conducted. The FEM and the experimental results were found to be in good agreement. The authors concluded that the response frequency increases when the height of the arch, the thickness of the front plate, the prestress of the front plate or the contact stiffness of the sound post increases. In a recent noteworthy study [72], the design variations of a Titian Stradivari violin were explored via FEM modeling. The model is parametric; hence, its design and material properties varied in both the frequency and time domains. The violin soundbox model included the top plate with the bass bar and the f-holes, the bridge, the back plate, the ribs, the blocks and the sound post, while the strings, neck and fingerboard were omitted. Moreover, two mass-spring-damper oscillators were considered and attached to the bass bar to incorporate the influence of vibrating components which are not included in the model. The well-known signature modes of the soundbox were observed, namely the A0, C-bout rhomboid (CBR), B_1^- and B_1^+. In Figure 5 are shown representative results of the simulated signature modes. The numerical results of the modal frequencies are found to be in good agreement with the experimental results found in the literature [73].

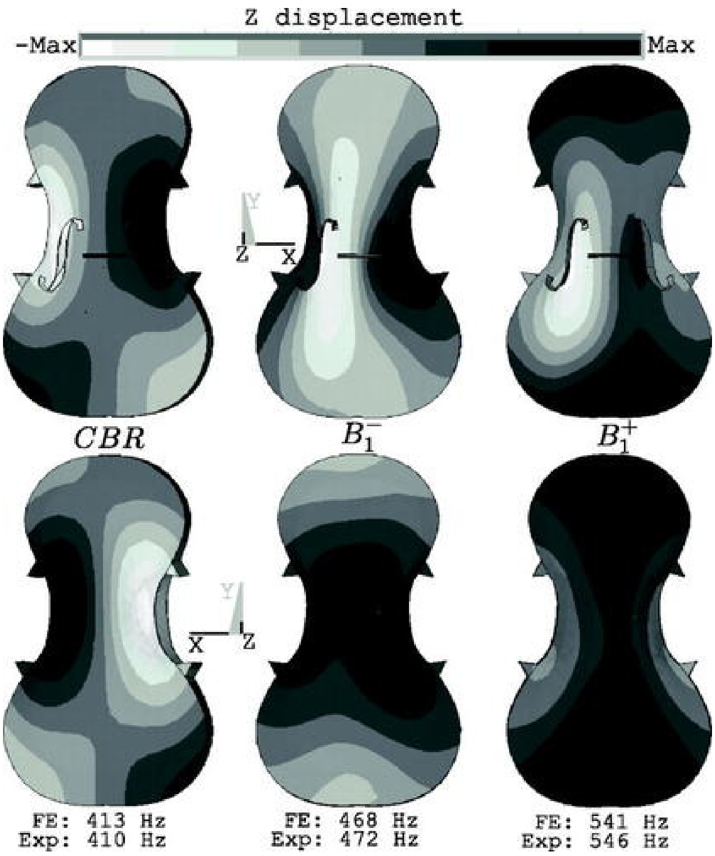

Figure 5. Simulated signature modes of the FE model, where the first row is for the top plate while the second row is for the back plate. Videos and 3D files of each simulated mode are available to be compared to the corresponding modes of Titian animated through the Polytec Scan Viewer; required data and software are included on the Strad3D DVD [73]. Images used are courtesy of ANSYS, Inc. Canonsburg, PA, USA. Reprinted with permission from [72]. Copyright 2020 Acoustic Society of America.

The cello is a bowed musical instrument which is less studied in the literature. Viala et al. [74] recently performed an eigenmode FEM analysis on an antique cello, concluding that the geometric accuracy of the developed model highly affects the accuracy of the simulation results of the antique cello model. This study concludes that the numerical models can simulate the effect of the restorer's and instrument maker's decisions on the instrument's final dynamical behavior. This can be a starting point for a decision support numerical tool that would be highly applicable to the conservation, maintenance and development of musical instruments.

5.2. Plucked Musical Instruments

Most of the studies on the plucked stringed instruments are focused on the guitar, and traditional instruments are less studied. Pedrammehr et al. [75] studied the vibration of the setar, a long-necked lute-type plucked Persian musical instrument, via FEM modeling and simulations. The 3D CAD model of the setar assembly was created via 3D laser scanning. The simulated mode shapes and natural frequencies of the setar structure were validated with experimental modal testing. A notable study by Mansour [76] analyzed the vibrational behavior of the setar by means of FEM and experimental modal analysis. A coordinate measuring machine (CMM) was used to define accurately the geometry of the soundbox. The developed FEM model considered structural details, such as the orthotropic properties of the wood, the direction of the grains, the nonideal joints and the effect of the preload of the strings. The CAD assembly of the soundbox of the setar is shown in Figure 6. The experimental analysis was performed using a combination of impulse hammer and laser Doppler vibrometer. In Figure 7, the comparison of representative experimental and numerical results of the mode shapes and the natural frequencies is demonstrated. The numerical results were found to be in good agreement with the experimental measurements over a wide range of frequencies.

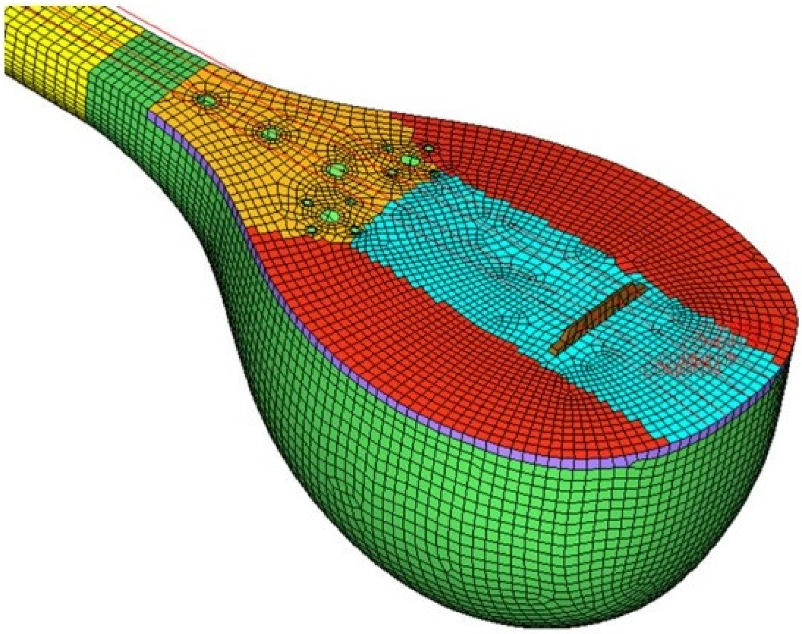

Figure 6. Assembly of the soundbox where the different shades represent element groups with different thicknesses. Reprinted with permission from [76]. Copyright 2015 American Society of Mechanical Engineers (ASME).

Regarding guitar studies, Patil et al. [77] performed an FEM analysis to determine an acoustic guitar's natural frequencies and mode shapes, validated by an experimental modal impact hammer test. Fleischer [78] studied the vibrational behavior of an electric bass guitar via FEM analysis and experimental laser scanning vibrometry. The bridge of a well-made solid-body bass was found to be less mobile than the neck. In contrast to the acoustic bass guitar, the fingerboard and the nut of the electric bass were found to be the terminations of greater vibrations.

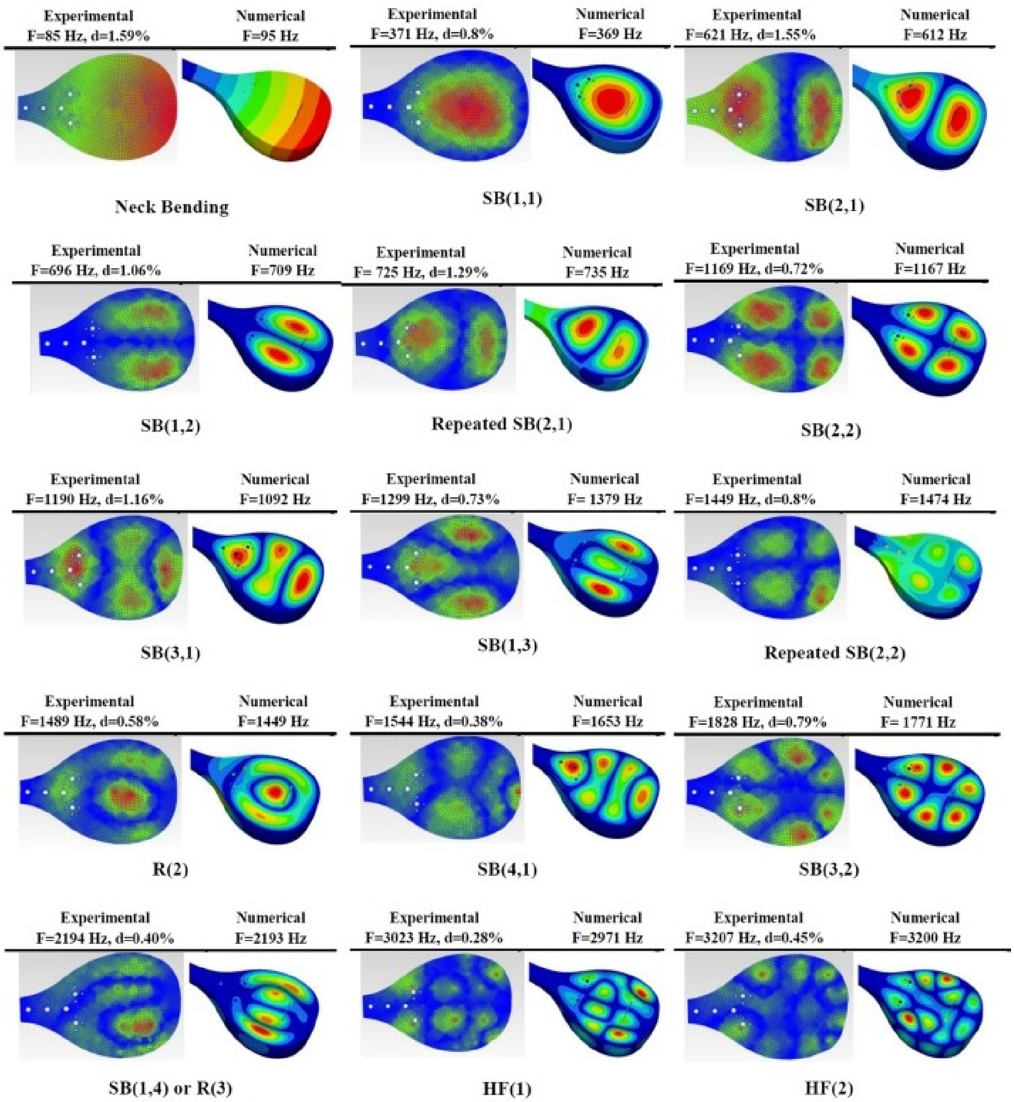

Figure 7. Mode shapes and natural frequencies of the plate obtained from the experimental and numerical results, compared together. The setar was clamped on its neck. Reprinted with permission from [76]. Copyright 2015 American Society of Mechanical Engineers (ASME).

6. FEM Fluid–Structure Interaction Studies of Assembled String Musical Instruments

This section presents FEM fluid–structure interaction studies, where the modeling of the resonance box of the instrument includes not only the wooden structure, but also the air inside, which is modeled as having fluid properties. The governing equation of the acoustic behavior of a confined acoustic fluid volume in the frequency domain is the Helmholtz equation for time-harmonic waves:

$$\nabla^2 p + k^2 p = 0 \tag{3}$$

where p is the acoustic pressure and k is the wavenumber. For vibroacoustic problems, the boundary condition is described by the following:

$$\frac{\partial p}{\partial n} = -i\rho\omega v_n \tag{4}$$

where n is the normal vector that points outside from the acoustic volume, ρ is the acoustic fluid density and v_n is the normal velocity.

Most literature studies concern the guitar because it is a slightly less complex mechanical system compared to the violin. Similar to many string instruments, the guitar resonance box has its air cavity connected to the outside by a hole, so that its dynamic behavior interacts with the external environment. The lowest air cavity mode, which is not coupled with the structure, is called the Helmholtz resonance and is denoted as A0. The higher modes of the air cavity (named A1, A2, etc.) correspond to the stationary waves inside it and are not harmonically related to A0 [79].

Elejabarrieta et al. [79] used the FEM on the air cavity of a guitar box, pointing out the effect of the sound hole. This preliminary model considered the air fluid as being confined in a rigid cavity with an orifice, the sound hole. The computed natural frequencies of the guitar air cavity were compared to those of experimental works from the literature [80–82], and good agreement was found. The same authors [83] studied the coupled modes of the resonance box–air cavity system. The modeled wooden resonance box comprised the top plate, the back plate, the bridge, the ribs, the edges and the blocks. The developed numerical model allowed one to study independently the modal analysis of the wooden structure and of the air inside the box, as well as the coupled modes of the resonance box–air cavity system. An experimental modal analysis technique was applied to validate the numerical results. The same research team [84] performed a numerical and experimental study where the interior gas varied. Natural frequencies, modal patterns and quality factors were determined when the box was full of either helium, air or krypton. The developed numerical fluid–structure model [83] was applied to study the coupled modes of the three different cases for the three different-in-density gases. One main conclusion was that the type of fluid determines the modal patterns and frequencies of the whole resonance box. In Figure 8, representative results of the calculated and measured coupled modes are demonstrated. It can be observed that the experimental and calculated modes are similar and appear in the same order. Furthermore, the authors concluded that the upper zone of the resonance box remains motionless, regardless of the type of gas in the frequency range analyzed. The literature also includes works that investigate the simulation of the fluid–structure interaction of the Brazilian [85] and the Portuguese [86] guitar resonance boxes (full 3D geometry). These results were validated against results from an experimental modal analysis.

Moreover, a noteworthy study on vibroacoustic modeling and simulation of the resonator of a Sarasvati veena, a South Indian stringed instrument, was recently performed by Chauhan et al. [87]. The Sarasvati veena is a plucked string wooden lute, whose unique timbre is characterized by the presence of nearly all harmonics, in the frequency range of 0 to 2800 Hz. An FEM numerical fluid–structure modal analysis was performed on a CAD model of the resonator top plate as well as the air cavity within the dome-shaped structure. The numerical results were compared to experimental modal analysis results of the veena resonator. In Figure 9, characteristic FEM and experimental results of the mode shapes of the top plate are demonstrated. A good agreement was observed for the modes presented in Figure 9a,c,h,j, while the modes presented in Figure 9f,g,k show a mismatch. To investigate this mismatch, sound pressure radiated by a resonator for unit applied force on the top plate was measured experimentally. Low pressure amplitudes were measured for the frequencies where the mismatch between FEM and modal experimental results was found. The authors concluded that this may justify the poor agreement for some of the modes.

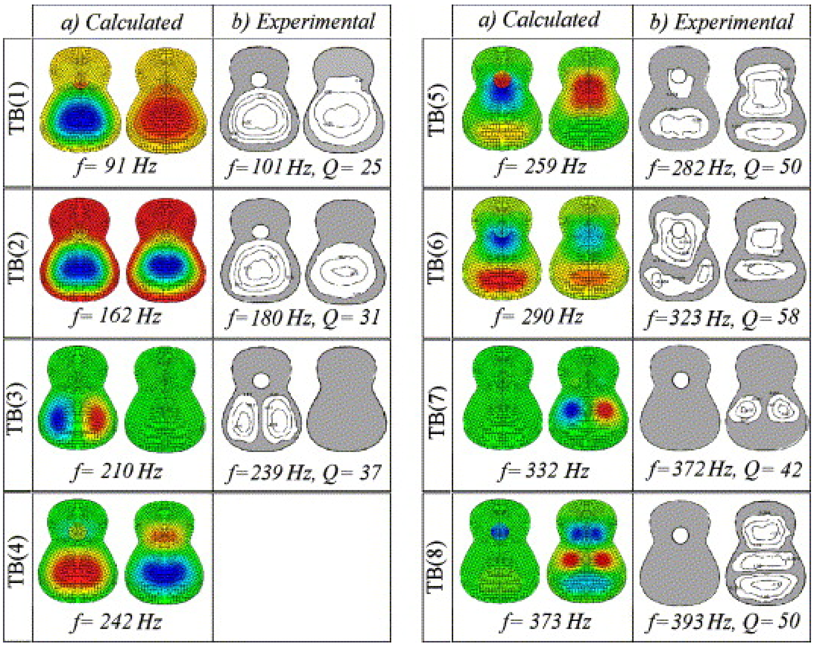

Figure 8. (**a**) Calculated and (**b**) experimental coupled modes corresponding to the box full of air. Reprinted with permission from [84]. Copyright 2005 Elsevier.

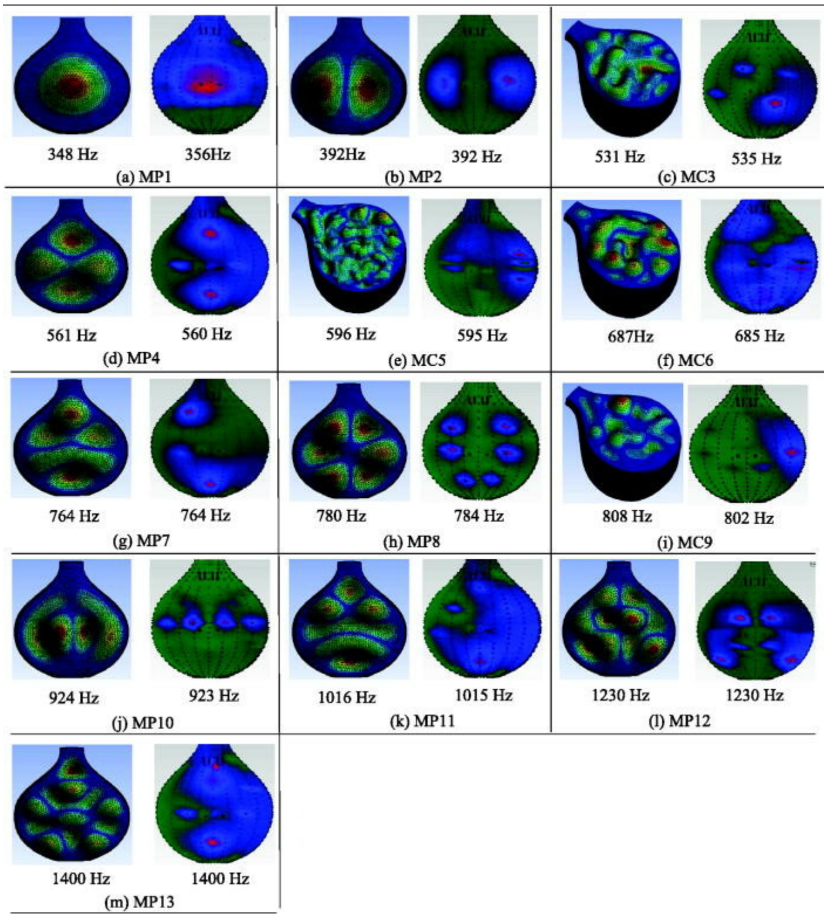

Figure 9. Comparison of FEM and experimental results of mode shapes (**a**–**m**) obtained with bottom shell fixed boundary condition. MP modes refer to the top plate and MC modes refer to the cavity alone. Reprinted with permission from [87]. Copyright 2021 Acoustic Society of America.

7. FEM Studies of String Musical Instrument Resonance Box Interaction with the Surrounding Air

Numerical studies of musical instrument box interaction with the surrounding air are relatively more limited in the literature due to the very high computational needs. An FEM–BEM preliminary study of guitar sound radiation was conducted by Brooke and Richardson [88]. The structural mode shapes were determined via FEM, while the resultant acoustic radiation was calculated via BEM. The computed radiation fields were compared with measurements on real systems, providing satisfactory agreement. Pyrkosz and Karsen [89] developed a vibroacoustic FE model for the Titian Stradivari violin. The structural model was made from actual CT scans of the actual violin. To handle the interior/exterior acoustic problem, an automatically matched layer (AML) property was considered. This feature constructs the absorbing layer on the solver level by extruding the boundary surface elements automatically [89]. The numerically predicted structural modes were correlated with experimental data of a modal analysis, while the acoustic results were compared to experimental sound radiation measurements. The vibroacoustic behavior of the Titian Stradivari violin was accurately predicted. The computed structural mode frequencies differed less than 5% compared to the experimentally measured modes. The computed main acoustic mode (A0) of the Titian Stradivari differed less than 7% compared to the experimental result. Recently, numerical simulations of the vibration mode of a violin body and the sound radiated by it were performed via FE analysis carried out by Yokoyama [90]. A microcomputed tomography scanner was used to create the instrument geometry. The eigenmodes and sound radiation were calculated via an acoustic–structure interaction module. Chatziioannou [91] developed an FE model of a reconstructed viola da gamba. The calculated modal shapes were validated with the aid of Chladni patterns and the ESPI experimental technique. The vibroacoustic simulation of the interaction between the vibrating surfaces of the instrument and the surrounding air demonstrated that the posited asymmetric instrument design may radiate sound more efficiently than a design involving a symmetric top plate. Figure 10 shows representative input admittance and acoustic efficiency results for the asymmetric and symmetric designs. It can be observed that the mobility of the bridge is similar for both geometries and is slightly higher in the symmetric case. However, the sound radiation is clearly higher for the asymmetric design over the simulated range of frequencies.

Although a lot of progress has been made in the field, from the literature results presented in Sections 4–7, it is evident that a detailed assembly including all the parts of the full instrument geometry resonance box, the fluid–structure interaction and the interaction with the surrounding air has not yet been developed and simulated. To the best of our knowledge, a primary effort by adopting simplified approximations has been conducted by Mansour et al. [92] for the setar more than 10 years ago. Although the modeling of the soundboards of stringed musical instruments using FEM was initiated in the 1980s, the modeling of the fluid–structure interaction was initiated in the 2000s and the vibroacoustic simulations were evolved significantly in the 2010s, the main reason that a full model has not yet been accomplished may be attributed to the fact that only a few research groups work exclusively on the study of stringed musical instruments globally. It is of crucial importance that the researchers validate their simulation results with experimental measurements, such as modal analysis and measurements of radiated sound pressure. However, this is not always the case. The synergy of simulations and experimental measurements is necessary to investigate thoroughly the complicated physical process of sound production on stringed musical instruments and to obtain accurate results. Additionally, the needs for computational resources to simulate such a complete computational assembly model are extremely high, and the use of an efficient high-performance computer (HPC) is mandatory.

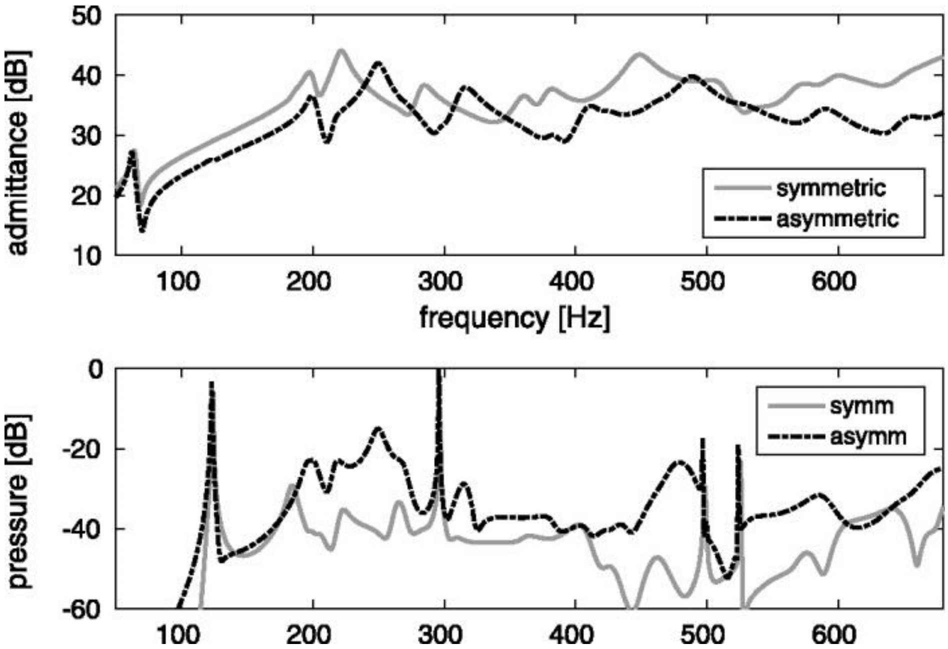

Figure 10. Magnitude of the viola bridge admittance (**top**) and acoustic efficiency (**bottom**) for a symmetric and an asymmetric instrument. Reprinted with permission from [91]. Copyright 2019 Acoustic Society of America.

8. Conclusions

The main scope of this review is to present the state of the art of stringed musical instrument FEM studies, categorized as follows: soundboard behavior, assembled musical instrument box, fluid–structure interaction and resonance box interaction with the surrounding air. From the 1980s to the present, numerical simulations based on FE models have evolved to become valuable tools for improving our understanding of the fundamental physical processes that occur in stringed musical instruments.

Most of the published research work is focused on the vibrating behavior of the soundboards of the stringed instruments. Although the modal frequencies of the free top plates are not directly related to the acoustic properties of the complete assembled instrument, the soundboard's modal frequencies are crucial parameters that decisively affect its final performance, according to the manufacturers. The numerical results are usually validated by experimental modal measurements. The violin is the most well studied bowed musical instrument. However, the physics of the bowed strings combined with the violin's complex geometry and material structure leads to many complications that are still of high research interest. Another common difficulty of the modeling process stems from the fact that accurate values of numerous different material parameters, such as Young's moduli, density and shear moduli, need to be included in the numerical models.

The modeling of the whole body of a musical stringed instrument is a much more complicated process that demands high computational resources. Moreover, due to the increase in the number of the modeled, parts the uncertainty of the values of the material properties increases significantly. For FEM studies of fluid–structure interactions of the complete instrument assembly, most studies found in the literature in the last two decades concern the guitar, which is a slightly less complex mechanical system compared to the violin. In the last decade, studies for lute-type stringed instruments such as the setar and the Sarasvati veena emerged.

Moreover, numerical studies of the musical instrument box interaction with the surrounding air are relatively more limited in the literature due to the extremely high computational needs. The numerical results are usually validated by measurements of radiated sound pressure. An FEM–BEM study of the sound radiated from a soundboard of a musical instrument is a highly time-consuming task, and the difficulties are even greater if the whole assembly and the fluid–structure interactions are also modeled. Thus, a detailed 3D FEM model including all the parts of the instrument's geometry, the resonance box and the air enclosure, also being surrounded by air and able to compute the fluid–structure interactions, has not yet been developed. The FEM modeling and the simulation of such a highly accurate and precise model is the future goal. The synergy of simulations and experimental measurements is more than necessary to investigate thoroughly the complicated physical processes of sound production on stringed musical instruments to obtain accurate results and achieve the goal.

Author Contributions: All authors of this research E.K., M.B., M.T., N.A.P. and V.D. contributed equally to its review and editing. Conceptualization, E.K.; writing—original draft preparation, E.K. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Sachs, C. *The History of Musical Instruments*; W. W. Norton & Company: New York, NY, USA, 1940.
2. Fletcher, N.; Rossing, T. *The Physics of Musical Instruments*; Springer: New York, NY, USA, 1998.
3. Benade, A.H. *Fundamentals of Musical Acoustics*; Oxford University Press: New York, NY, USA, 2012.
4. Bucur, V. *Handbook of Materials for String Musical Instruments*; Springer: Cham, Switzerland, 2016.
5. Giordano, N. Some observations on the Physics of stringed instruments. In *Springer Handbook of Systematic Musicology*; Springer: Berlin/Heidelberg, Germany, 2018; pp. 105–119.
6. Bader, R.; Hansen, U. Modeling of Musical Instruments. In *Handbook of Signal Processing in Acoustics*; Springer: New York, NY, USA, 2008; pp. 419–446.
7. Bader, R. Characterizing Classical Guitars Using Top Plate Radiation Patterns Measured by a Microphone Array. *Acta Acust. United Acust.* **2011**, *97*, 830–839. [CrossRef]
8. McIntyre, M.E.; Woodhouse, J. The acoustics of stringed musical instruments. *Inter. Sci. Rev.* **1978**, *3*, 157–173. [CrossRef]
9. Rossing, T.D. Modal analysis. In *Springer Handbook of Acoustics*; Springer: New York, NY, USA, 2007; pp. 1127–1138.
10. Jones, R.; Wykes, C. *Holographic and Speckle Interferometry*; Cambridge University Press: Cambridge, UK, 1989.
11. Castellini, P.; Revel, G.M.; Tomasini, E.P. Laser Doppler vibrometry: Development of advanced solutions answering to technology's needs. *Mech. Syst. Signal. Process.* **2006**, *20*, 1265–1285. [CrossRef]
12. Wang, C.P. Laser Doppler velocimetry. *J. Quant. Spectrosc. Radiat. Transfer.* **1988**, *40*, 309–319. [CrossRef]
13. Bakarezos, E.; Orphanos, Y.; Kaselouris, E.; Dimitriou, V.; Tatarakis, M.; Papadogiannis, N.A. Laser-based interferometric techniques for the study of musical instruments. In *Computational Phonogram Archiving*; Springer: Cham, Switzerland, 2019; pp. 251–268.
14. Bakarezos, E.; Vathis, V.; Brezas, S.; Orphanos, Y.; Papadogiannis, N.A. Acoustics of the Chelys—An ancient Greek tortoise-shell lyre. *Appl. Acoust.* **2012**, *73*, 478–483. [CrossRef]
15. Herrero-Durá, I.; Picó, R.; Sánchez-Morcillo, V.J.; Garcia-Raffi, L.M. Modelling of continuous elastic systems by using the Finite Element Method. *Model. Sci. Educ. Learn.* **2017**, *10*, 193–202. [CrossRef]
16. Klímová, H.; Tippner, J. Modal analysis of soundboard of the upright piano by Finite Element Method (FEM). *Wood Res.* **2014**, *59*, 123–136.
17. Zienkiewicz, O.C.; Taylor, R.L. *The Finite Element Method*; McGraw Hill Book Company: London, UK, 1967.
18. Hughes, J.R. *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*; Dover Publications: Mineola, NY, USA, 1987.
19. Kaselouris, E.; Nikolos, I.K.; Orphanos, Y.; Bakarezos, M.; Papadogiannis, N.A.; Tatarakis, M.; Dimitriou, V. Elastoplastic study of nanosecond-pulsed laser interaction with metallic films using 3D multiphysics fem modeling. *Int. J. Damage Mech.* **2016**, *25*, 42–55. [CrossRef]
20. Kaselouris, E.; Nikolos, I.K.; Orphanos, Y.; Bakarezos, E.; Papadogiannis, N.A.; Tatarakis, M.; Dimitriou, V. A review of simulation methods of laser matter interactions focused on nanosecond laser pulsed systems. *J. Multiscale Model.* **2013**, *5*, 1330001. [CrossRef]
21. Bilbao, S. Conclusion and perspectives. In *Numerical Sound Synthesis: Finite Difference Schemes and Simulation in Musical Acoustics*; John Wiley & Sons: Hoboken, NJ, USA, 2009; pp. 386–388.
22. Välimäki, V.; Pakarinen, J.; Erkut, C.; Karjalainen, M. Discrete-time modelling of musical instruments. *Rep. Prog. Phys.* **2006**, *69*, 1–78. [CrossRef]
23. Chauhan, C.; Singru, P.; Vathsan, R. The effect of the extended bridge on the Timbre of the Sarasvati Veena: A numerical and experimental study. *J. Meas. Eng.* **2021**, *9*, 23–35. [CrossRef]
24. Bader, R. Physical model of a complete classical guitar body. In Proceedings of the Stockholm Music Acoustics Conference, Stockholm, Sweden, 6–9 August 2003; Volume 1, pp. 21–124.

25. Bader, R. *Computational Mechanics of the Classical Guitar*; Springer: Berlin/Heidelberg, Germany, 2005.
26. Moosrainer, M.; Fleischer, H. Application of BEM and FEM to musical instruments. In *Boundary Elements in Acoustics*; Advances and Applications; WIT Press: Southampton, UK, 2000.
27. Richardson, B. Numerical modeling of stringed musical instruments. In *SMAC 93*; Friberg, A., Ed.; Royal Swedish Academy of Music: Stockholm, Sweden, 1993; pp. 457–462.
28. Kaselouris, E.; Alexandraki, C.; Orphanos, Y.; Bakarezos, M.; Tatarakis, M.; Papadogiannis, N.A.; Dimitriou, V. Acoustic analysis of impact sound on vibrating circular membranes. In Proceedings of the INTER-NOISE 2021—2021 International Congress and Exposition of Noise Control Engineering, Washington, DC, USA, 1–4 August 2021; Institute of Noise Control Engineering: Reston, VA, USA, 2021; Volume 63, pp. 3378–3385.
29. Schleske, M. Empirical Tools in Contemporary Violin Making: Part I. Analysis of Design, Materials, Varnish, and Normal Modes. *Catgut Acoust. Soc. J.* **2002**, *4*, 50–64.
30. Hutchins, C.M. A history of violin research. *J. Acoust. Soc. Am.* **1983**, *73*, 1421–1440. [CrossRef]
31. Woodhouse, J. The acoustics of the violin: A review. *Rep. Prog. Phys.* **2014**, *77*, 115901. [CrossRef]
32. Hutchins, C.M.; Benade, V. (Eds.) *Research Papers in Violin Acoustics, 1975–1993*; Acoustical Society of America: Washington, DC, USA, 1996; Volumes 1 & 2.
33. Knott, G.A.; Shin, Y.S.; Chargin, M.A. Modal analysis of the violin. *Finite Elem. Anal. Des.* **1989**, *5*, 269–279. [CrossRef]
34. Bretos, J.; Santamaria, C.; Moral, J.A. Vibrational patterns and frequency responses of the free plates and box of a violin obtained by finite element analysis. *J. Acoust. Soc. Am.* **1999**, *105*, 1942–1950. [CrossRef]
35. Rodgers, O.; Anderson, P. Finite element analysis of violin corpus. *Catgut Acoust. Soc. J.* **2001**, *4*, 13–26.
36. Gough, C.E. A violin shell model: Vibrational modes and acoustics. *J. Acoust. Soc. Am.* **2015**, *137*, 1210–1225. [CrossRef]
37. Marshall, K.D. Modal analysis of a violin. *J. Acoust. Soc. Am.* **1985**, *77*, 695–709. [CrossRef]
38. Jansson, E.; Molin, N.-E.; Saldner, H.O. On eigen modes of the violin—Electronic holography and admittance measurements. *J. Acoust. Soc. Am.* **1994**, *95*, 1100–1105. [CrossRef]
39. Rossing, T.D.; Molin, N.-E.; Runnemalm, A. Modal analysis of violin bodies viewed as three-dimensional structures. *J. Acoust. Soc. Am.* **2003**, *114*, 2438. [CrossRef]
40. Bissinger, G. Structural acoustics of good and bad violins. *J. Acoust. Soc. Am.* **2008**, *124*, 1764–1773. [CrossRef] [PubMed]
41. Hutchins, C.M. The acoustics of violin plates. *Sci. Am.* **1981**, *245*, 170–186. [CrossRef]
42. Rubin, C.; Farrat, D.F. Finite element modelling of violin plate vibrational characteristics. *J. Catgut Acoust. Soc.* **1987**, *47*, 8–11.
43. Rodgers, O. The effect of elements of wood stiffness on violin plate vibration. *J. Catgut Acoust. Soc.* **1988**, *1*, 2–8.
44. Schumacher, R.T. Compliances of wood for violin top plates. *J. Acoust. Soc. Am.* **1988**, *84*, 1223–1235. [CrossRef]
45. Yu, Y.; Jang, I.G.; Kim, I.K.; Kwak, B.M. Nodal line optimization and its application to violin top plate design. *J. Sound Vib.* **2010**, *329*, 4785–4796. [CrossRef]
46. Stanciu, M.D.; Coşereanu, C.; Dinulică, F.; Bucur, V.T. Effect of wood species on vibration modes of violins plates. *Eur. J. Wood Wood Prod.* **2020**, *78*, 785–799. [CrossRef]
47. Lu, Y. Comparison of Finite Element Method and Modal Analysis of Violin Top Plate. Master's Thesis, McGill University, Montréal, QC, Canada, 2013.
48. Molin, N.E.; Lindgren, L.-E.; Jansson, E.V. Parameters of violin plates and their influence on the plate modes. *J. Acoust. Soc. Am.* **1988**, *83*, 281–291. [CrossRef]
49. Richardson, B.E.; Roberts, G.W.; Walker, G.P. Numerical modelling of two violin plates. *J. Catgut Acoust. Soc.* **1987**, *47*, 12–16.
50. Gough, C. Violin plate modes. *J. Acoust. Soc. Am.* **2015**, *137*, 139–153. [CrossRef]
51. Curtin, J. Tap tones and weights of old Italian violin tops. *J. Violin Soc. Am.* **2005**, *20*, 161–174.
52. Gonzalez, S.; Salvi, D.; Baeza, D.; Antonacci, F.; Sarti, A. A data-driven approach to violin making. *Sci. Rep.* **2021**, *11*, 9455. [CrossRef] [PubMed]
53. Gonzalez, S.; Salvi, D.; Antonacci, F.; Sarti, A. Eigenfrequency optimisation of free violin plates. *J. Acoust. Soc. Am.* **2021**, *149*, 1400–1410. [CrossRef] [PubMed]
54. Kaselouris, E.; Orphanos, Y.; Bakarezos, M.; Tatarakis, M.; Papadogiannis, N.A.; Dimitriou, V. Influence of the plate thickness and material properties on the violin top plate modes. In Proceedings of the INTER-NOISE 2021—2021 International Congress and Exposition of Noise Control Engineering, Washington, DC, USA, 1–4 August 2021; Institute of Noise Control Engineering: Reston, VA, USA, 2021; Volume 263, pp. 3369–3377.
55. Wilczyński, T.J.; Filipek, R.; Wilczyński, P. Investigation of influence of pre-stresses on Viola da Gamba sound using FEM. In Proceedings of the 7th Forum Acusticum, FA 2014, Krakow, Poland, 7–12 September 2014.
56. Bakarezos, M.; Gymnopoulos, S.; Brezas, S.; Orphanos, Y.; Maravelakis, E.; Papadopoulos, C.I.; Tatarakis, M.; Antoniadis, A.; Papadogiannis, N.A. Vibration analysis of the top plates of traditional greek string musical instruments. In Proceedings of the 13th International Congress on Sound and Vibration 2006, ICSV 2006, Vienna, Austria, 2–6 July 2006; Volume 6, pp. 4939–4946.
57. Elejabarrieta, M.J.; Ezcurra, A.; Santamaría, C. Evolution of the vibrational behavior of a guitar soundboard along successive construction phases by means of the modal analysis technique. *J. Acoust. Soc. Am.* **2000**, *108*, 369–378. [CrossRef] [PubMed]
58. Skrodzka, E.; Lapa, A.; Linde, B.B.J.; Rosenfeld, E. Modal parameters of two incomplete and complete guitars differing in the bracing pattern of the soundboard. *J. Acoust. Soc. Am.* **2011**, *130*, 2186–2194. [CrossRef]

59. Torres, J.A.; Boullosa, R.R. Influence of the bridge on the vibrations of the top plate of a classical guitar. *Appl. Acoust.* **2009**, *70*, 1371–1377. [CrossRef]
60. Ezcurra, A. Influence of the material constants on the low frequency modes of a free guitar plate. *J. Sound Vib.* **1996**, *194*, 640–644. [CrossRef]
61. Torres, J.A.; De Icaza, M.; Boullosa, R.R. Variations in Mechanical Properties of Wood Plates Due Fluctuations on Relative Humidity of Air Mater. *Res. Soc. Symp. Proc.* **2012**, *1*, 139–144.
62. Shepherd, M.R.; Hambric, S.A.; Wess, D.B. The effects of wood variability on the free vibration of an acoustic guitar top plate. *J. Acoust. Soc. Am.* **2014**, *136*, EL357. [CrossRef]
63. Salvi, D.; Gonzalez, S.; Antonacci, F.; Sarti, A. Modal analysis of free archtop guitar top plates. *J. Acoust. Soc. Am.* **2021**, *150*, 1505–1513. [CrossRef] [PubMed]
64. Viala, R.; Pérez, M.A.; Placet, V.; Manjón, A.; Foltête, E.; Cogan, S. Towards model-based approaches for musical instruments making: Validation of the model of a Spanish guitar soundboard and characterization features proposal. *Appl. Acoust.* **2021**, *172*, 107591. [CrossRef]
65. Kindel, J.; Wang, I.-C. Modal analysis and finite element analysis of a piano soundboard. In Proceedings of the 5th International Modal Analysis Conference (IMAC), London, UK, 6–9 April 1987; pp. 1545–1549.
66. Berthaut, J.; Ichchou, M.N.; Jézéquel, L. Piano soundboard: Structural behavior, numerical and experimental study in the modal range. *Appl. Acoust.* **2003**, *64*, 1113–1136. [CrossRef]
67. Moore, T.R.; Zietlow, S.A. Interferometric studies of a piano soundboard. *J. Acoust. Soc. Am.* **2006**, *119*, 1783–1793. [CrossRef] [PubMed]
68. Mamou-Mani, A.; Frelat, J.; Besnainou, C. Numerical simulation of a piano soundboard under downbearing. *J. Acoust. Soc. Am.* **2008**, *123*, 2401–2406. [CrossRef] [PubMed]
69. Ege, K.; Boutillon, X.; Rébillat, M. Vibroacoustics of the piano soundboard: (Non)linearity and modal properties in the low- and mid-frequency ranges. *J. Sound Vib.* **2013**, *332*, 1288–1305. [CrossRef]
70. Corradi, R.; Miccoli, S.; Squicciarini, G.; Fazioli, P. Modal analysis of a grand piano soundboard at successive manufacturing stages. *Appl. Acoust.* **2017**, *125*, 113–127. [CrossRef]
71. Zhang, C.-Z.; Ye, B.-Y.; Hu, X.-Z.; Zhao, X.-Z.; Cao, Y.-F. Finite element simulation application for vibration characteristic of violin. *Key Eng. Mater.* **2014**, *620*, 662–667. [CrossRef]
72. Torres, J.A.; Soto, C.A.; Torres-Torres, D. Exploring design variations of the Titian Stradivari violin using a finite element model. *J. Acoust. Soc. Am.* **2020**, *148*, 1496–1506. [CrossRef]
73. Zygmuntowicz, S. The Strad 3D project: Scientists, musicians, and violinmakers study three classic violins. *J. Acoust. Soc. Am.* **2010**, *127*, 1791. [CrossRef]
74. Viala, R.; Placet, V.; Le Conte, S.; Vaiedelich, S.; Cogan, S. Model-Based decision support methods applied to the conservation of musical instruments: Application to an antique cello. In *Model Validation and Uncertainty Quantification*; Conference Proceedings of the Society for Experimental Mechanics Series; Springer: Cham, Switzerland, 2020; pp. 223–227.
75. Pedrammehr, S.; Aghdam, N.J.; Pakzad, S.; Ettefagh, M.M.; Sadeghi, M.H. A study on vibration of Setar: Stringed Persian musical instrument. *J. Vibroeng.* **2018**, *20*, 2680–2689. [CrossRef]
76. Mansour, H. Modal Analysis of the Setar: A Numerical-Experimental Comparison. *J. Vib. Acoust. Trans. ASME* **2015**, *137*, 61006. [CrossRef]
77. Patil, K.; Baqersad, J.; Ludwigsen, D.; Dong, Y. Extracting vibration characteristics of a guitar using finite element, modal analysis, and digital image correlation techniques. *Proc. Meet. Acoust.* **2016**, *29*, 065003.
78. Fleischer, H. Vibration of an electric bass guitar. *Acta Acust. United Acust.* **2005**, *91*, 247–260.
79. Elejabarrieta, M.J.; Santamaría, C.; Ezcurra, A. Air cavity modes in the resonance box of the guitar: The effect of the sound hole. *J. Sound Vib.* **2002**, *252*, 584–590. [CrossRef]
80. Jansson, E.V. Acoustical properties of complex cavities prediction and measurements of resonance properties of violin-shaped and guitar-shaped cavities. *Acustica* **1977**, *37*, 211–221.
81. Rossing, T.D.; Popp, J.; Polstein, D. Acoustical response of guitars. In *SMAC 83*; Royal Swedish Academy of Music: Stockholm, Sweden, 1985; pp. 311–332.
82. Runnelmalm, A.; Molin, N.E. Air cavity modes in sound boxes recorded by TV holography. *J. Acoust. Soc. Am.* **1999**, *105*, 1125. [CrossRef]
83. Elejabarrieta, M.J.; Ezcurra, A.; Santamaria, C. Coupled modes of the resonance box of the guitar. *J. Acoust. Soc. Am.* **2002**, *111*, 2283–2292. [CrossRef]
84. Ezcurra, A.; Elejabarrieta, M.J.; Santamaría, C. Fluid-structure coupling in the guitar box: Numerical and experimental comparative study. *Appl. Acoust.* **2005**, *66*, 411–425. [CrossRef]
85. Paiva, G.O.; Dos Santos, J.M.C. Modeling fluid-structure interaction in a Brazilian guitar resonance box. In Proceedings of the 22nd International Congress of Mechanical Engineering (COBEM 2013), Ribeirão Preto, SP, Brazil, 3–7 November 2013.
86. Vieira, M.; Infante, V.; Serrão, P.; Ribeiro, A.M.R. Experimental-numerical correlation of the dynamic behavior of the Portuguese guitar. *Appl. Acoust.* **2018**, *131*, 51–60. [CrossRef]

87. Chauhan, C.; Singru, P.M.; Vathsan, R. Vibro-acoustic modeling, numerical and experimental study of the resonator and its contribution to the timbre of Sarasvati veena, a South Indian stringed instrument. *J. Acoust. Soc. Am.* **2021**, *149*, 540–555. [CrossRef] [PubMed]
88. Brooke, M.; Richardson, B.E. Numerical modeling of guitar radiation fields using boundary elements. *J. Acoust. Soc. Am.* **1991**, *89*, 1878. [CrossRef]
89. Pyrkosz, M.A.; Van Karsen, C. Coupled vibro-acoustic model of the Titian stradivari violin. In *Topics in Modal Analysis I*; Conference Proceedings of the Society for Experimental Mechanics Series; Springer: Berlin/Heidelberg, Germany, 2014; Volume 7, pp. 317–332.
90. Yokoyama, M. Coupled numerical simulations of the structure and acoustics of a violin body. *J. Acoust. Soc. Am.* **2021**, *150*, 2058–2064. [CrossRef] [PubMed]
91. Chatziioannou, V. Reconstruction of an early viola da gamba informed by physical modeling. *J. Acoust. Soc. Am.* **2019**, *145*, 3435–3442. [CrossRef] [PubMed]
92. Mansour, H.; Kasaiezadeh, A.; Arzanpour, S.; Behzad, M. Finite element modeling of setar, a stringed musical instrument. In Proceedings of the ASME 2009 International Mechanical Engineering Congress and Exposition, Sound, Vibration and Design, Lake Buena Vista, FL, USA, 13–19 November 2009; Volume 15, pp. 591–597.